Training Module

Mathematics

Class X

$$(x-3)(x-2)=0$$
$$\Rightarrow x-3=0 \text{ or, } x-2=0$$

$$(x-3)(x-2)=2$$
$$\nRightarrow x-3=2 \text{ or, } x-2=2$$

West Bengal Board of Secondary Education
Department of School Education, Govt. of West Bengal
Samagra Shiksha Abhiyan
Planning and Development : Expert Committee,
Department of School Education

Training Module
MATHEMATICS

Class X

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Department of School Education
Govt. of West Bengal

Bikash Bhavan, Kolkata - 700091

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July, 2020

The Teachers' Training Programme under SSA will be conducted according to this module that has been developed by the Expert Committee on School Education and approved by the WBBSE.

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FROM THE BOARD

In 2011 the Honourable Chief Minister Smt. Mamata Banerjee constituted the Expert Committee on School Education of West Bengal. The Committee was entrusted upon to develop the curricula, syllabi and textbooks of the school level of West Bengal. The Committee therefore had developed school textbooks from Pre-Primary level, Class I to Class VIII based on the recommendations of National Curriculum Framework (NCF) 2005 and Right to Education (RTE) Act 2009. In 2016 the new curriculum and syllabus of 'Ganit' for Class X came into effect and textbooks were developed accordingly. However, certain questions evoke in our minds: (i) How will the competencies of the learners modified, refined or improved in Class X? (ii) How far can the learners establish themselves as citizens of value and responsibility at the end of Class X? (iii)How far can the learners go beyond the limits of academic disciplines to apply knowledge in their social life? And in trying to find suitable answers for these questions the Expert Committee developed the framework of the Constructivist methodology for knowledge construction.

Following the recommendations of Samagra Shiksha Abhiyan (SSA), the Govt. of West Bengal has arranged an orientation programme of 'Ganit' for Class X on the method of learning and evaluation. This 'Training Module' has been developed for the said orientation programme.

The Hon'ble Minister in Charge for Education, Dr. Partha Chatterjee, has enriched with his views and comments. We express our sincerest gratitude to him.

We hope that the orientation programme will be successful and have a lasting effect in the teaching-learning process of the future.

Kalyanmoy Ganguly

President
West Bengal Board
of
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July, 2020
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PREFACE

The Honourable Chief Minister Smt. Mamata Banerjee constituted the Expert Committee on School Education of West Bengal in 2011. The Committee was given the responsibility to review, reconsider and reconstitute all the aspects of the school curriculum, syllabi and textbooks. The new curriculum, syllabi and textbooks were developed based on the recommendations of the Expert Committee.

The school textbooks for all classes, from Pre-Primary level to Class VIII, were developed following the guidelines of NCF 2005 and RTE Act 2009. The textbooks for Class X were developed based on the new curriculum and syllabus.

Following the recommendations of Samagra Shiksha Abhiyan (SSA), the Govt. of West Bengal has organized an orientation programme on the method of learning and evaluation of 'Ganit' for Class X. This 'Training Module' has been developed for the said orientation programme.

The Hon'ble Minister in Charge for Education, Dr. Partha Chatterjee, has enriched us with his views and comments. We express our gratitude to him.

The State level Teachers' orientation programme on the methodology of learning and evaluation has been planned and executed in assistance with School Education Department, Govt. of West Bengal, West Bengal Board of Secondary Education and Samagra Shiksha Abhiyan (SSA). It is hoped that the 'Training Module', developed on behalf of School Education Department, Govt. of West Bengal, West Bengal Board of Secondary Education and Samagra Shiksha Abhiyan (SSA), will help in the effective implementation of the methodology of learning and evaluation.

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Samagra Shiksha Abhiyan (SSA)

Introduction

The Right of Children to Free and Compulsory Education (RTE) Act, 2009, seeks to ensure that children enjoy the benefits of the three aspects of Access, Equity and Quality in school education across the nation. To this effect, the Ministry of Human Resource & Development (MHRD) in line with the proposal of the Union Budget, 2018 -2019 has initiated the scheme of SAMAGRA SHIKSHA ABHIYAN (SSA). The scheme takes a holistic stance in treating school education from Pre-Primary to Class XII as a continuum by merging the erstwhile Sarva Shiksha Abhiyan and Rashtriya Madhyamik Shiksha Abhiyan schemes in one, unified whole.

Scope of SSA

The Samagra Shiksha Abhiyan (SSA) collates the three Schemes of Sarva Shiksha Abhiyan, Rashtriya Madhyamik Shiksha Abhiyan and Teacher Education.The SSA scheme aims at improving school effectiveness measured in terms of equal prospects for schooling and equitable learning outcomes. In harmonizing the different and major effectual factors of school education, the SSA scheme provides for the operational mechanisms and transaction costs at all levels, particularly in using state, district and circle level systems and resources, besides envisioning one comprehensive strategic design for advancement of school education. The shift in the focus is from project objectives to refining systems level performance and schooling outcomes which will be the emphasis of the SSA scheme, alongwith encouraging States towards improving quality of education.

Major Objectives of SSA

The holistic nature of the scheme envisages Universal Access, Equity and Quality, promotion of Vocational Education, refurbishment of the use of Soft or e-Materials in schools and strengthening of Teacher Education.

The major objectives of the scheme are summarized below:

- **Provision of Quality Education and enhancing learning outcomes of students**
- **Bridging Social and Gender Gaps in School Education**
- **Ensuring Equity and Inclusion at all levels of School Education**
- **Ensuring minimum standards in schooling provisions**
- **Support States in implementation of Right of Children to Free and Compulsory Education (RTE) Act, 2009**

Features of the textbook *Ganit Prakash* (class X) and methodology of knowledge construction

- **Known to Unknown**

National Curriculum Framework (NCF) 2005 expects the learners to be able to correlate her/his life in school to that of the life beyond it. As s/he comes to the classroom s/he brings with her/him naïve concepts of her/his society. In the classroom s/he analyses the concepts with mathematical reasoning and reconstructs them with the help of the teacher. S/he, then, applies her reconstructed knowledge in real life.

Hence, it is anticipated that the Mathematics teachers will start a lesson keeping in view on this aspect of learning: knowledge acquired outside the classroom! reconstruction of knowledge in school! application of knowledge beyond the classroom.

For example, the learners might have heard about business, capital or sharing of profit etc. from real life situations beyond their school experience. They get a chance of analyzing the same experience within the classroom in the chapter on 'Partnership' and are able to understand how the profit is shared among the investors. The teacher also helps them whenever required. When they go beyond the classroom they are able to implement that concept in real life.

- **Child Centric Education**

In child-centric education, both the teacher and the child are engaged in activities. And to keep the child engaged in activities the teacher has to implement different types of Teaching Learning Materials (TLM). The objective is to facilitate each learner in knowledge construction, help them in innovative thinking and encourage them in diverse experimentation. For example, the teacher will help them to understand why the ratios of trigonometry are always on the angles of 30^0, 45^0 and 60^0. S/he will also teach them what instrument has to be used and how it can be manufactured.

- **Constructivist Learning**

Students should not learn anything through memorization without understanding or knowing the reason behind it. Knowledge should be gradually constructed by the synthesis of the information gathered from the society or nature and that provided by the teacher. For example, the students should be engaged to create problems that form quadratic equations.

- **Concrete to abstract**

In teaching Mathematics, if the process is from concrete to abstract, their conception becomes clear. Hence, the teacher should gradually lead the students from the concept of concrete to abstract. For example, the students are to be taught how to use 30^0, 45^0 and 60^0 angles of a right angled triangle to measure the height and distance before teaching them the definitions of trigonometric ratios for measuring any θ angle.

- **Activity Based Learning**

It is better to provide the basic idea of a topic through hands-on activities. Consequently, clear concept is developed by working out concrete activities in a joyful way. Provision of additional activities beyond the textbook facilitates the learners to construct knowledge joyfully and thereby discover abstract concepts embedded in the lesson.

- **Joyful learning**

Learning acquisition is possible when the students learn in joyful environment. The teacher might enter the classroom with proper lesson plan, hands-on activities etc. However, active participation of learners in the learning process is not possible until the classroom environment becomes joyful. Hence, it is anticipated that the learners will be able to develop their concept with much ease and comfort when the teacher makes the class environment joyful.

- **Integration of curricular and co-curricular activities**

Since quality enhancement of curricular and co-curricular activities has been recommended for all disciplines, it is hoped that the teachers will engage the learners in multifarious correlated activities on Mathematics (like drawing, drama, quiz, music, debate etc.) for knowledge construction.
Theorems of circle can be applied for lighting and sitting arrangement of audience in theater.

- **Peer Learning**

Students engaged in group activities in the classroom can develop their concept on a particular topic through group discussions. Often the learners forming knowledge at slow pace seek help from their peers who develop their concept fast. In fact mutual cooperation develops among the learners through group learning.

For example, in course of the discussion on trigonometric ratio, students may be divided into groups. They may be asked to draw three right angled triangles different measurement varying the angle at the base as 30^0, 45^0 and 60^0 respectively and thereby to find out the trigonometric ratios. Although different groups have made triangles of different measurement, trigonometric ratios of each group remain the same. Their findings would evidently surprise them. They would try to discover about the reason of their findings through discussions and the teacher would facilitate them when required.

- **Integration of different subjects**

Care has been taken that the learners are able to integrate different subjects and issues of the society with the different concepts that they have acquired in the various lesson of Mathematics. For example, the students have learnt how to solve the problems of Physical Science with trigonometric ratios. They have also learnt about the use of sphere while reading Geography.

- **Learning without Burden**

Education becomes a burden to learners when they are burdened with information and tested with textbook oriented examination. However, in the present scenario of learner centric education if students develop concept from their experience and apply it for solving problems in real life, education becomes joyful and learning becomes burden free.

A closer look at the Mathematics textbook for class X will reveal there is a conscious effort to help the students so that they can themselves understand mathematical concepts and construct their knowledge to apply them rationally solving different types of problems and even realize the importance of the concepts in real life. Many colourful diagrams and pictures are included to make the textbook attractive.

As children grow up, their sound knowledge at the Primary and Upper Primary level help them to construct knowledge expediently at the Madhyamik level. Keeping at par with this vision of education at the national level, the syllabus of Mathematics has been lessened at the Primary level and is gradually increased at the Upper Primary and Madhyamik level.

The new textbook of Mathematics for class X is not burdened with some data, complicated calculations or solving problems. Instead, the book contains some familiar chronicles of mathematics, hands-on activities, simple calculations to solve problems necessary in real life and thereby making the textbook burden free and enjoyable.

- **Quality Education**

Knowledge construction of the learners and its proper application should not be left incomplete in the classroom. It is hoped that the teacher would take care so that the learner would acquire the expected learning outcomes of each lesson. The expected learning outcomes of quadratic equation are given below:

The learners will be able to—

- construct quadratic equations in real life problems
- differentiate between quadratic equation and quadratic expression
- find out the coefficients of x^2, x^1, x^0 in a quadratic equation
- solve quadratic equations by the process of factorization and the rules of real numbers, i.e. to find the roots of the quadratic equation
- solve quadratic equations by the rule of Sridhar Acharya
- find the relation between the coefficients and roots of quadratic equations
- tell the type of roots without solving the quadratic equation
- calculate quadratic equation if the roots are known
- apply the concept of quadratic equation to solve different problems of real life

- **Equity**

There should not be any kind of discrimination (gender wise, political, economic, social, cultural, physical, psychological etc.) while presenting a story or problem on mathematics. The teacher should be careful about it.

- **Values**

The objectives of formal school education are to develop knowledge among the learners and also to make them socially responsible and true citizens of the nation. That is, s/he should not be a citizen severed from the society.

Hence, in various lessons of the textbook efforts have been made to develop values of different types through anecdotes like 'Tree planting programme' (minimizing environment pollution), 'Celebration of birth anniversary of famous personalities' (knowing about them and their advice to become honest, true citizen), 'Let's share food equally' (living alone is not life), 'Going to the market with grandfather' or 'Let's go to aunt's place' or Let's go to aunt's house or village house' (maintaining good social and family relationship) etc.

- **General idea of the textbook**

At the very outset of a lesson a topic is introduced in the learner's familiar environment. After that, some hands-on activities are provided to engage the learners actively to develop a basic idea of the lesson. Some problems of real life are also given so that they can solve the problems rationally using their experience of hands-on activities. Then the essential mathematical terms and symbols required for mathematical knowledge construction are also given at the right place. Gradually all the domains of learning in that lesson are taken care of so that the learners can easily and perfectly understand the topic to form their knowledge. Finally, problems on critical thinking are also given. It is to be noted that modern education is not result oriented: it is Process and Product based. Hence, in the textbook, emphasis has been given on process.

Thus, the book is an endeavor to develop concept through familiar ideas, realizes the necessity of the lesson, develops logical concept through hands- on activities, forms analytical skill by solving simple problems, develops logic by solving complex problems and also manifests absolute logic through abstract concepts.

Despite all these, the textbook is only an instrument of learning. It is not possible to make learner-centric education joyful for all students, in all schools, across the state with a single textbook. Unfortunately, in our country, textbook is considered to be the only means of education. ("The present day classroom practices are, in almost all schools of the country, totally dominated by textbook… The textbook emerges as the single solution to all these problems."—National Curriculum Framework 2005, Position paper 2.3, Article no. 4.2.5, page no. 37). Hence, it is hoped if the teachers of all schools prepare different types of TLMs (including mathematical stories) and various problems beyond the textbook, in respect of the quality of students of their respective schools, then teaching-learning would be truly joyful and the quality of education would be enhanced.

- **Support according to specific requirement and gradual withdrawal of support**

The textbook has been so designed that the learners can themselves construct knowledge and will take help from their teacher only when it is required.

- **Chronicles to introduce the lesson followed by hands-on activity**

If an abstract concept of mathematics is introduced at the very outset, often mathematics becomes a phobia for most learners. However, if a lesson of mathematics begins with a problem of real life, the learners are able to understand the necessity of the lesson. So mathematics ceases to become a phobia. It becomes a subject of great interest. Again, children are fond of playing games. Hence, they are involved in various hands-on activities to develop their knowledge through logic.

However, this does not mean that in classroom transaction a lesson should always be started with the anecdote or activity included in the textbook. It is expected that the teacher will start the lesson with a story or real life problem of her/his own considering the geographical or cultural position of the school and thus making the lesson extremely attractive and interesting.

- **Empty spaces in the pages of the textbook**

If the learner can read fluently and understand the contents of the textbook and is able to construct knowledge logically, s/he will be able to fill up the empty spaces of the book. Those spaces are provided to assess her/his gradual development of logical thinking. Besides, spaces are given so that the learner reads the textbook from the first page to the last. In fine, mathematics textbook does not necessarily imply a book to solve problems. The idea is to develop the concept of reading a mathematics book.

- **Different branches of mathematics not clubbed together**

In the textbook after some lessons on arithmetic there is a lesson on algebra, followed by a lesson on geometry, then statistics, and again a lesson on algebra.

Different branches of mathematics often have integration. For example, the lesson on Pythagoras theorem is followed by trigonometric ratio and then on trigonometric identities because the lesson is based on the previous lessons.

- **Multiple choice and short answer type questions**

At the end of every lesson Multiple choice questions (MCQ) and short answer type questions are provided so that students who can rationally construct the idea of the lesson will be able to solve the problems using the concept.

- **Development of Life Skill**

It is imperative to adjust oneself in adverse conditions of life and use the minimum opportunities embedded in the situation to overcome the problem of life thereby achieving quality education. Needless to say, there is great possibility and scope for the development of life skills of the students in respect of mathematics. Regular practice helps in the development of the learner. Hence, one of the key components of mathematics learning is the regular evaluation of the life skills.

Construction of Knowledge by addressing Naïve Concepts

The naïve concepts of the learners are to be addressed through classroom discussion helping them to understand their misconceptions. The entire process is to be conducted in three stages:

(i) **Catch:**

- The naïve concepts should not be corrected instantly. Learners are allowed to ask questions that are to be listened carefully.
- Learners are to be encouraged to discuss in groups. Their interactions should be heeded carefully.

(ii) **Challenge:**

- Logical explanations about their ideas are to be elicited.
- Data based interpretations are to be encouraged.

(iii) **Change:**

- Paradoxes are to be created in the learners about naïve concepts to develop true conception.
- Learners are to be encouraged to apply the correct method of solving problems based on the newly developed conception.
- Application of new conceptions for solving real life problems is to be encouraged.

For example, while solving a quadratic equation we generally factorize the algebraic expression on the left side. We can then easily find out the factors of the given equation. Students often develop some misconceptions in the later stages of such problems that lead them to errors. The teachers are expected to explain the reason of such conceptual errors to the learners.

Case no. 1: x^2-5x +6=0

Solve the quadratic equation:

$$
\begin{aligned}
& x^2 - 5x + 6 = 0 \\
\Rightarrow\ & x^2 - 3x - 2x + 6 = 0 \\
\Rightarrow\ & x(x-3) - 2(x-3) = 0 \\
\Rightarrow\ & (x-3)(x-2) = 0 \\
\Rightarrow\ & x - 3 = 0 \quad \text{or, } x - 2 = 0 \\
\Rightarrow\ & x = 3 \quad \text{or, } x = 2
\end{aligned}
$$

Case no.2: x^2-5x +6=2

Solve the quadratic equation:

$$
\begin{aligned}
& x^2 - 5x + 6 = 2 \\
\Rightarrow\ & x^2 - 3x - 2x + 6 = 2 \\
\Rightarrow\ & x(x-3) - 2(x-3) = 2 \\
\Rightarrow\ & (x-3)(x-2) = 2 \\
\Rightarrow\ & x - 3 = 2 \quad \text{or, } x - 2 = 2 \\
\Rightarrow\ & x = 2 + 3 \quad \text{or, } x = 2 + 2 \\
\Rightarrow\ & x = 5 \qquad \text{or, } x = 4
\end{aligned}
$$

In the second case, the method of solving the problem is erroneous. The teachers are expected to explain the students about the cause of the error. They will help the learners to form proper conception by using real numbers.

Syllabus

1. Quadratic equation in one variable

i) Concept of quadratic equation in one variable
ii) Concept of quadratic equation in one variable $ax^2+bx+c=0$ (a,b,c are real numbers and $a\neq0$)
iii) Solution of quadratic equation with the help factorization. (Roots are rational numbers.)
iv) Solution of quadratic equation by expressing perfect square.
v) Concept of Sridhara Acharyya's formula.
vi) Concept about the nature of roots.
vii) Concept of construction of a quadratic equation in one variable if roots are known.
viii) Solution of real problems of quadratic equation in one variable.

2. Simple Interest

i) Concept of principal, interest, rate of interest in percent per annum, amount, time.
ii) Concept of the formula ($I = \frac{prt}{100}$)
iii) Concept of solution of different real problems.

3. Theorems related to circle.

i) In the same circle or in equal circles, equal chords intercept equal arcs and subtend equal angels at the centre (Proof is not necessary)
ii) In the same circle or in equal circles, the chords which subtend equal angles at the centre are equal (proof is not necessary).
iii) One and only one circle can be drawn through three non-collinear points. (Proof is not necessary)
iv) If a line drawn from the centre of any circle bisects the chord, which is not a diameter, will be a perpendicular on the chord— proof.
v) A perpendicular drawn from the centre of a circle on a chord, which in not a diameter, bisects the chord - proof.
vi) Application of above statements.

4. Rectangular Parallelopiped or Cuboid

i) Concept of the things of the shape of retanglular parallelopiped and cube which are seen in real life.
ii) Concept of the number of the surfaces, edges, vertices and diagonals.
iii) Concept of formation of formula of total surface area.
iv) Concept of formation of formula of volume.
v) Concept of formation of formula of the length of a diagonal.
vi) Concept of solution of different real problems.

5. Ratio and proportion

i) Concept of ratio and proportion in Algebra.
ii) Concept of different types of ratio and proportion
iii) Concept of application of different proportional properties in the problems related to proportion

6. Compound Interest (upto 3 years) and uniform rate of increase or decrease

i) Concept of difference in simple interest and compound interest.

ii) Concept of formation of formula if the compound interest is given yearly, half-yearly and quarterly.

iii) Concept of solution of different real problems.

iv) Concept of formula formation of uniform rate of increase or decrease from the formula of compound interest.

v) Concept of solution of real problems.

7. Theorems related to angles in a circle

i) Concept of angle subtended at the centre and in the circle

ii) The angle subtended at the centre by an arc is twice that of an angle subtended in the circle– proof

iii) In any circle, angles in the same segment are equal—proof.

iv) Angle in a semicircle is a right-angle — proof.

v) If a straight line segment makes equal angles at the two points situated on the same side of it, then the four points are concylic. (proof is not necessary)

vi) Application of above statements.

8. Right Circular Cylinder

i) Concept of right circular cylinders which are seen in real life.

ii) Concept of curved surface and plane surface of a right circular cylinder.

iii) Concept of formula formation of curved surface area.

iv) Concept of formula formation of total surface area.

v) Concept of formula of volume.

vi) Concept of solution of real problems of different types.

9. Quadratic Surd

i) Concept of irrational numbers.

ii) Concept of quadratic Surds.

iii) Concept of pure, mixed, like and unlike quadratic Surds

iv) Concept of rationalising factor

v) Concept of rationalising factor of denominator.

vi) Concept of addition, subtraction, multiplication and division of quadratic surds.

vii) Concept of solution of different real problems of quadratic surds.

10. Theorems related to cyclic quadrilateral

i) The opposite angles of a cyclic quadrilaterals are supplementary to each other– proof

ii) If the opposite angles of a quadrilateral are supplementary to each other, then the vertices of quadrilateral are concyclic– (Proof is not necessary).

iii) Application of above statements.

11. Construction : Construction of circumcircle and incircle of a triangle.

i) Construction of circumcircle of a given triangle.

ii) Construction of incircle of a given triangle.

iii) Construction of a circle about a given triangle (proof is not included in Evaluation)

12. Sphere

i) Concept of a solid with the shape of sphere and hemisphere which are seen in real life.

ii) Concept of surfaces of sphere and hemisphere.

iii) Concept of curved surface area of a sphere

iv) Concept of curved surface area and total surface area of a hemisphere.

v) Concept of volumes of sphere and hemisphere.

vi) Concept of solution of different real problems.

13. Variation

i) Concept of simple variation, inverse variation and compound variation.

ii) Concept of different problems related to variation, inverse variation and solution of real problems.

14. Partnership Business

i) Concept about partnership business

ii) Concept of simple and mixed partnership business.

iii) Concept about principal.

iv) Concept of distribution of dividend

v) Application of ratio in different real problems related to partnership business.

15. Theorems related to Tangent to a circle.

i) Concept of tangent and transversal of a circle.

ii) The tangent and the radius passing through the point of contact are perpendicular to each other — proof

iii) If two tangents are drawn from an external point, then the two line segments joining external point and point of contact are equal and they make equal angles at the centre— proof.

iv) Concept of direct common tangent and transverse common tangent.

v) If two circles touch each other, then two centres of two circles and point of contact are collinear– proof.

vi) Application of above statements.

16. Right circular cone.

i) Concept of right circular conical solids which are seen in real life.

ii) Concept of curved surface and plane surface of a right circular cone.

iii) Concept of curved surface area of a right circular cone.

iv) Concept of total surface area of a right circular cone.

v) Concept of volume of a right circular cone.

vi) Solution of different real problems.

17. Construction : Construction of tangent to a circle.

i) Concept of construction of tangent of a circle to a point on the circle.

ii) Concept of construction of two tangents to a circle from an cxternal point.

18. Similarity

i) Concept of similar geometric figures.

ii) A line drawn parallel to any side of a triangle divides other two sides or extended two sides proportionally (proof is not, necessary.)

iii) If any straight line divides two sides or extended two sides of a triangle proportionally, then the straight line will be parallel to third side. (proof is not necessary)

iv) If two triangles are similar, their corresponding sides are proportioal (proof is not necessary)

v) If the sides of two triangles are proportional then their corresponding angles are equal. (proof is not necessary)

vi) In two triangles, if an angle of one is equal to an angle of the other and the adjacent sides of the angles are proportional, then the two triangles are similar. (proof is not necessary)

vii) If in a right angled triangle, a perpendicular is drawn from its angular point to its hypotenuse, then the two triangles obtained are similar with original triangle and they are similar to each other– proof

viii) Applications of above statements.

19. Problems related to different soild objects.

i) Solution of real problems related to different soild objects (rectangular parallelopiped, right circular cylinder, sphere, hemisphere, right circular cone)

20. Trigonometry : concept of measurement of angle.

i) Evolution, growth and explanation of necessity of trigonometry in reality.

ii) Concept of positive and negative angles.

iii) Concept of measurement of angle.

iv) Concept of sexagesimal system and circular system, concept of their relations and application in different problems.

21. Construction : Determination of mean proportional.

i) Determination of mean proportional of two line segments in geometric method.

ii) Construction of a square whose area is equal to a rectangle.

iii) Construction of a square with area equal to a triangle.

22. Pythagoras theorem

i) Pythagoras theorem – proof.

ii) Converse of Pythagoras theorem – proof.

iii) Applications of above theorem.

23. Trigonometric Ratios and Trigonometric Identities.

i) Concept of different trigonometric ratios with respect to a right angled triangle.

ii) Concept of relations among different trigonometric ratios.

iii) Determination of the values of trigonometric ratios of some standard angles (0º, 30º, 45º, 60º, 90º) and concept of applications in different problems.

iv) Concept of applications of trigonometic ratios in different problems.

v) Concept of elimination of an angle (viz. θ) from trigonometric ratios.

24. Trigonometric Ratios of complementary angle

i) Concept of complementary angle.

ii) Concept of trigonometric ratios of a complementary angle of an angle and concept of solution of different problems.

25. Application of Trigonometric Ratios : Heights and Distances

i) Concept of angle of elevation and angle of depression.

ii) Concept of solution of real problems by trigonometric method with the help of right angled triangle, angle of elevation and angle of depression.

26. Statistics : Mean, Median, Ogive, Mode.

i) Concept of measures of central tendency.

ii) Concept of average or mean.

iii) Concept of three methods for determination of mean (a) direct method, (b) short method (c) standard deviation.

iv) Concept of needs of determination of median.

v) Concept of the formula require to determine median and concept of solution of different real problems.

vi) Concept of cumulaive frequency curved line or ogive.

vii) Concept of determination of median from ogive.

viii) Necessity for determination of mode.

ix) Concept of determination of formula for mode and concept of solution of different real problems.

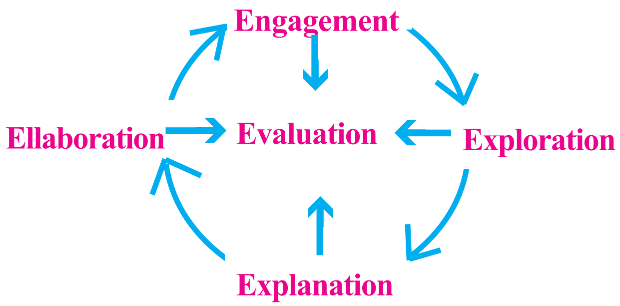
x) Concept of relations among mean, median and mode.

Constructivist Approach in classroom transaction

Selected learning areas, knowledge construction and Lesson Framework

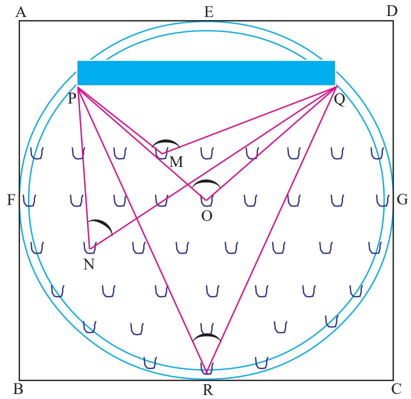
Topic: **Theorem on Angle subtended at a point on the Circle**

Learning Methodology: 5E Model

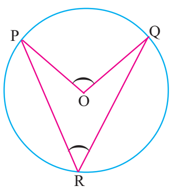


Engage: [Generate Interest]

ABCD is a square plain surface. EFRG is a circular area within the plain surface. The stage and the seats are in the same surface within the circular area. The points in the diagram ($\cup$) are the seats. PQ is the front stage. We will try to find out, at what point the angle is created in the circuler region that is double the angle created by PQ on a seat at the point R. We will then measure the angles of each seat created by PQ.



For example, $\angle PRQ = 60^\circ$, $\angle PMQ = 130^\circ$, $\angle PNQ = 110^\circ$ and $\angle POQ = 120^\circ$

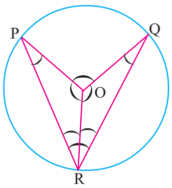


Explore [Establish relationship]

We were surprised to there is no angle, except the one angle at the centre, that is double the angle of $\angle$PRQ.

Explain [Communicate new understanding]

We will try to understand the reason of our findings. We joined O and R.



Let, $\angle OPR = \theta \therefore \angle ORP = \theta$

$\therefore \angle POR = 180^\circ - 2\theta$

Let, $\angle OQR = \alpha \therefore \angle ORQ = \alpha$

$\therefore \angle QOR = 180^\circ - 2\alpha$

$\therefore \angle POQ = 360^\circ - (180^\circ - 2\theta + 180^\circ - 2\alpha)$

$= 2(\theta + \alpha) = 2\angle PRQ$

$\therefore \angle PRQ = \frac{1}{2} \angle POQ$

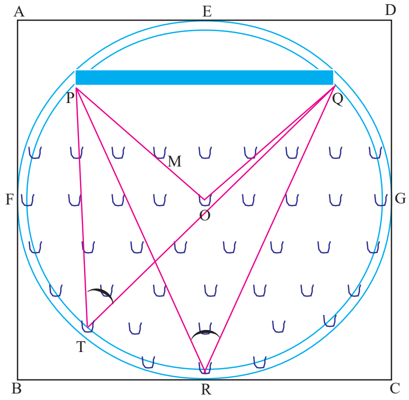
Here we have used two known areas of our learning:

(i) The measurement of the opposite angles of the two equal sides of an isosceles triangle are equal.

(ii) The sum of the measurement three angles of a triangle is equal to two right angles.

Extend/Elaborate
[Apply new learning to a new or similar situation]

When we tried to measure the angle created by PQ on any given point like R within the circle, we were surprised to find that—



$\angle PRQ = 60^o = \angle PTQ$

Let us try to prove the above findings

We have seen if

$\angle POQ = x^o$, then $\angle PRQ = \frac{1}{2} x^o$

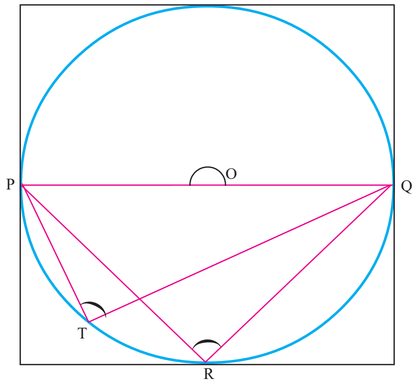
In the same way, we can prove that

$\angle PTQ = \frac{1}{2} x^o$

Or, $\angle PRQ = \angle PTQ$

Evaluate
[Apply the knowledge to real life problem]

Suppose the stage covered half of the circular area and the seats the other half. Then, we try to measure the angles created at R of the front stage and the seat at T. We find that both the angles are right angles. We are evidently surprised. How does it happen? Let us try to give logical explanation.



We have noticed that in this case $\angle POQ$ is a straight angle.

That is, $\angle POQ = 180^0$

Hence, $\angle PTQ = \frac{1}{2} \times 180^o = 90^o$

Similarly, $\angle PRQ = 90^o$

Math Lab Activity

Topic: Finding relation through activities between the angle subtended by an arc at the centre and the angle subtended at a point on the circle by the same arc

Previous knowledge:

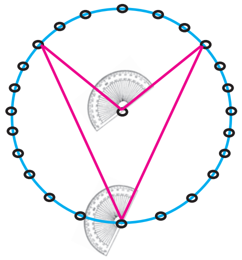
- circle, centre of the circle, arc of the circle
- angle subtended by an arc at the centre of the circle
- angle subtended by an arc at the circle

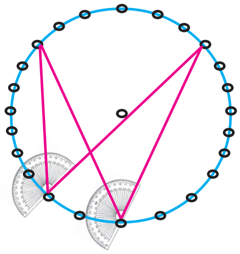
Learning objectives:

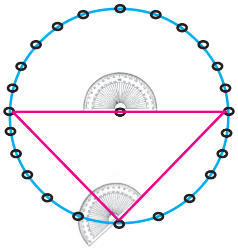
- to know the properties of the angle subtended by an arc at a point on the circle
- to verify about the concept that the angle subtended at the centre of the circle is double the angle subtended at a point on the circle on the same arc.
- to apply the concept of angle subtended at the circle on different geometrical proofs.

Learning materials: Cardboard, glue, white paper, materials of a geometry box, small pins, garter

Methodology:







- A circle of any radius is drawn on the white paper. Suppose the centre of the circle is O. Some points at equidistance are marked within the circle.
- The paper is glued to the cardboard.
- Small pins are fixed to the cardboard at each point and also at the centre.
- A garter is attached to the pins to make a subtended angle at the centre. Another garter is used to make a subtended angle in the circle within the same arc of the circle.
- The subtended angles at the centre and in the circle are measured with a protractor.
- It is found that subtended angle by an arc at the centre is double the subtended angle in the circle.
- All the angles subtended by the same arc at the circle are equal.
- Angle of semicircle formed by the garter is measured by a protractor.

Thus, we found that

(i) the angle subtended by an arc at the centre is double the angle subtended at a point on the circle on the same arc of the circle.

(ii) The angles subtended at the circle on the same arc are equal.

(iii) The angle subtended by semicircle is equal to one right angle.

Worksheet: 1

A. Choose the correct answer:

(i) If the vertex of an angle is at the centre of the circle, then it is the angle

(a) subtended by semicircle (b) subtended at the circle

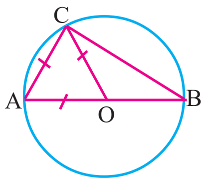
(c) subtended at the centre (d) subtended by semicircle or subtended at the circle

(ii) If any two chords intersect at a point on the circle, the angle that is formed is called

(a) angle subtended at the centre (b) reflexive angle

(c) angle subtended by semicircle or at the circle (d) angle subtended at the centre or reflexive angle

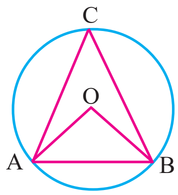
(iii) In the given figure if A, B, C are the three points on the circle with centre O and “AOC is an equilateral triangle, then $\angle BCO$ is equal to



(a) ½ $\angle AOB$ (b) ½ $\angle ACB$

(c) ½ $\angle AOC$ (d) ½ $\angle COB$

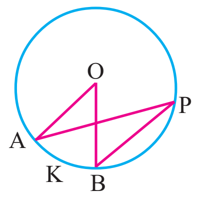
(iv) In the given figure if A, B, C are the three points on the circle with centre O. ABC is an isosceles triangle and $\angle BAC = 50^0$, then the value of $\angle AOC$ is



(a) 80^0 (b) 100^0

(c) 160^0 (d) 50^0

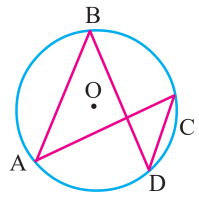
B. Fill in the blanks:



(i) The angle subtended by the arc AKB at the circle in the given figure is___________.

(ii) In the figure with question (i) the angle subtended by the arc AKB at the centre is____________.

(iii) In the figure with question (i) if $\angle AOB = 80^\circ$, then the value of $\angle APB$ is ____________.



(iv) In the adjacent figure, ifABD=50º, then the value of $\angle ACD$ is_____________.

C. Write True/False:

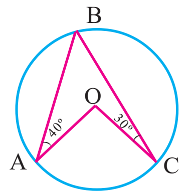
(i) The angle of semicircle is the angle subtended at the centre.

(ii) The opposite angles of a cyclic quadrilateral are always equal to each other.

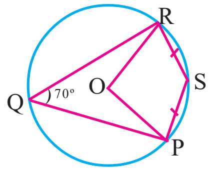
(iii) All the four angles of a cyclic quadrilateral are angles subtended at the circle.

(iv) The arcs of equal measure always form equal angles at the centre of the circle.

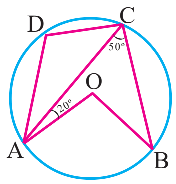
D. Answer the following questions:



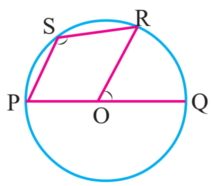
(i) In the adjacent figure if $\angle OAB = 40^\circ$ and $\angle OCB = 30^\circ$ of the circle with centre O, then find the value of $\angle AOC$.



(ii) In the adjacent figure P, Q, R, S are four points on the circle with centre O and PORS is a quadrilateral. PS=RS and $\angle PQR = 70^0$. Find the value of $\angle ORS$.



(iii) In the adjacent figure, A, B, C and D are four points on the circle with centre O. If $\angle ACB = 50^\circ$ and $\angle OAC = 20^\circ$, find the value of $\angle ADC$.

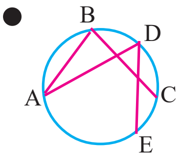
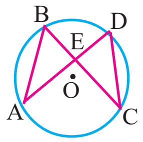
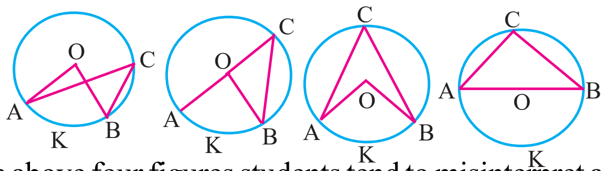
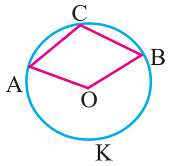
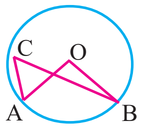


(iv) In the adjacent figure, PQ is the diameter of the circle with centre O. S and R are two points on the circle. Find the relation of $\angle PSR$ and $\angle QOR$.

E. (i) Apply the theorem that the angle subtended by an arc at the centre is double the angle subtended at the circle and prove that the opposite angles of a cyclic quadrilateral are supplementary with each other.

(ii) Prove that the mid-point of the hypotenuse of a right angled triangle is equidistant from the three vertices.

Based on the activities given above some examples of possible misconceptions or errors of the students are given below:

Possible errors of the students	Remedial
•  In the figure both the angles ∠ABC and ∠ADE are the angles in the same segment.	• The two angles are not in the same segment because ∠ABC is on the segment ABDC of the circle and ∠ADE is on the segment ADCE of the circle.
•  $\angle ABC = \frac{1}{2}\angle AEC$ and $\angle ADC = \frac{1}{2}\angle AEC$	• ∠AEC is not an angle subtended at the centre because the vertex of the angle subtended at the centre is on the centre of the circle.
•  In the above four figures students tend to misinterpret as they cannot understand the relation between the measure of the angle subtended at the centre and the angle subtended at the circle.	The relation between the measures of the angle subtended by an arc AKB at the centre and the angle subtended at the circle are the same due to various position of the point C on the arc.
•  The sum of the measurement of the opposite angles of the quadrilateral AOBC is 180º.	AOBC is not a cyclic quadrilateral because in a cyclic quadrilateral all the four vertices are on the circle.
•  ∠ACB= ½ ∠AOB	∠ACB is not an angle subtended at the circle because the vertex of the angle is not on the circle.
Students know that if the angle subtended at the circle is 90^0 then the line segment that connects the two end points of the sides is the diameter of the circle. But they do not know that the angle will always be angle of semicircle.	The angle of semicircle is always 90^0 and the chord that forms the angle is always the diameter of the circle.

Expected Learning Outcomes:

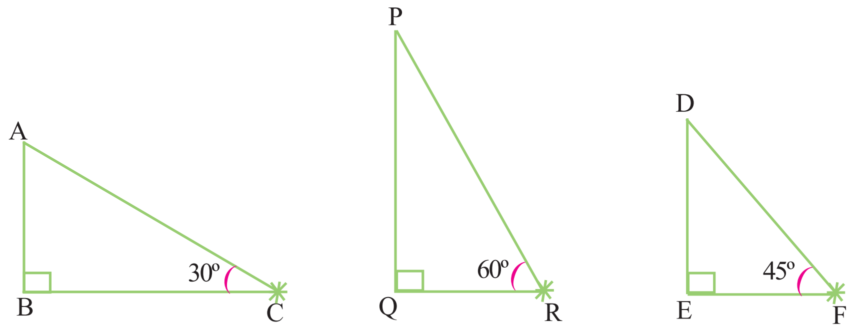
The learner will be able to—

- Recognize the angles subtended by an arc at the centre and at the circle.
- Apply the concepts, like the measurements of two angles of an isosceles triangle are equal, the sum of the measurements of three angles of a triangle is 180° and the sum of all the angles of around a point is 360°, in the theorems on angles subtended by an arc at the circle
- Develop relation between the angles subtended at the centre and at a point on the circle in the same arc
- Develop relation between the angles subtended by the arc at the centre with the angles subtended by the same arc at a point on the circle
- Infer about the relation between the angles subtended at the points on the circle
- Establish that angle subtended at a point on the semicircle is a right angle
- Apply the theorems on angle subtended at a point on the circle to solve various problems of geometry

Topic: Trigonometric ratio

Learning methodology: **ICON Model**

Previous knowledge: **Concept of ratio, concept of Pythagoras theorem**



Observation: Three big trees of the school playground fell one after another by the squall but the top of broken parts were not completely separated from the trunks. Each of the broken parts touched the ground. It seems that the length of broken part of the first tree is almost double the length of the remaining part. The length of broken part of the second tree is double the distance from the base to the top. The length of the remaining portion of the third tree is equal to the distance from the base to the top.

Again, it is observed that in each case, the highest point of each tree created three different angles in relation to its base.

Contextualisation

Is there any relation between the measure of the angles and the length of the sides?

Cognitive Apprenticeship

The students will be told to sit into groups and each group will draw three right angled triangles so that"

I. The length of the hypotenuse of the first triangle is double that of the perpendicular

II. The length of the hypotenuse of the second triangle is double that of the base

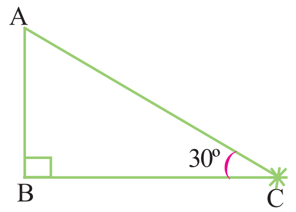
III. The length of the perpendicular of the third triangle is equal to that of the base

In all the three triangles the angle between the base and the hypotenuse are to be measured.

They will be told to find relation between the angles and the respective sides of the triangle. The teacher will help them whenever required.

Students will discuss in groups and try to solve the problem.

Collaboration



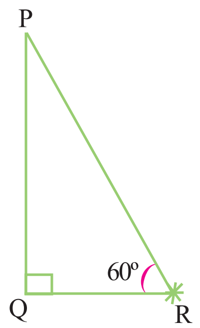
In the first case, "ABC is a right angled triangle, where AC=2.AB. Measuring, the angle $\angle BCA = 30^\circ$. Interestingly, all the groups found the measure of that angle to be 30º. They will know that in respect of $\angle BCA$, the perpendicular is AB and the base is BC. Thus, the ratio of the two sides of the triangle, in respect of that angle, can be expressed. This ratio can be given a name. For instance"

Length of the perpendicular/ Length of the hypotenuse

$= \sin \angle BCA = AB/AC = AB/2.AB = ½$

$\Rightarrow \sin 30^\circ = ½$

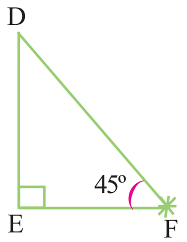
This is known as Trigonometric Ratio.



In the second case, ΔPQR is a right angled triangle where PR=2.QR. Measuring the angle PRQ =60º. Interestingly, all the groups found the measure of that angle to be 60º. They will know that in respect of $\angle PRQ$, the perpendicular is PQ and the base is QR. Thus, the ratio of the two sides of the triangle, in respect of that angle, can be expressed. This ratio can be given a name. For instance"

$$\frac{\text{length of the base}}{\text{length of the hypotenuse}} = \cos \angle PRQ = \frac{QR}{PR} = \frac{QR}{2.QR} = \frac{1}{2}$$

$$\Rightarrow \cos 60^\circ = \frac{1}{2}$$



In the third case, ΔDEF is a right angled triangle, where DE=EF. Measuring, the angle DFE =45°. Interestingly, all the groups found the measure of that angle to be 45°. They will know that in respect of $\angle$DFE, the perpendicular is DE and the base is EF. Thus, the ratio of the two sides of the triangle, in respect of that angle, can be expressed. This ratio can be given a name. For instance

$$\frac{\text{Length of the perpendicular}}{\text{Length of the base}} = \tan \angle DFE = \frac{DE}{EF}$$

$\Rightarrow \tan 45^\circ = 1.$

This is also known as Trigonometric Ratio.

Multiple interpretation

So, we have found three trigonometric ratios. Taking any two sides of the triangle, six ratios can be formed. Hence, there can be six trigonometric ratios.

In the case of 30° angle, $\frac{\text{length of the hypotenuse}}{\text{length of the perpendicular}} = \operatorname{cosec} \angle BCA$

$\therefore \operatorname{cosec} 30^\circ = \frac{2\,AB}{AB} = 2$

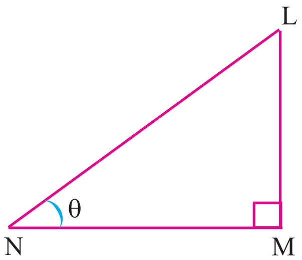
In the case of 60° angle, $\frac{\text{length of the hypotenuse}}{\text{length of the base}} = \sec \angle PRQ$

$\therefore \sec 60^\circ = \frac{2\,QR}{QR} = 2$

In the case 45° angle, $\frac{\text{length of the base}}{\text{length of the perpendicular}} = \cot \angle DFE$

$\therefore \cot 45^\circ = \frac{EF}{DE} = \frac{DE}{DE} = 1$

Thus, in a right angled triangle LMN, $\angle LMN = 90^\circ$ and $\angle MNL = \theta$ [suppose the unknown angle adjacent to base is θ]



$$\sin\theta = \frac{\text{length of the perpendicular}}{\text{length of the hypotenuse}} = \frac{LM}{NL}$$

$$\cos\theta = \frac{\text{length of the base}}{\text{length of the hypotenuse}} = \frac{MN}{NL}$$

$$\tan\theta = \frac{\text{length of the perpendicular}}{\text{length of the base}} = \frac{LM}{MN}$$

$$\operatorname{cosec}\theta = \frac{\text{length of the hypotenuse}}{\text{length of the perpendicular}} = \frac{NL}{LM}$$

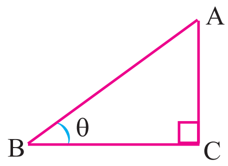
$$\sec\theta = \frac{\text{length of the hypotenuse}}{\text{length of the base}} = \frac{NL}{MN}$$

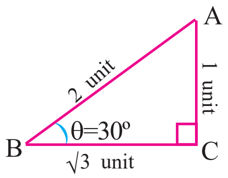
$$\cot\theta = \frac{\text{length of the base}}{\text{length of the perpendicular}} = \frac{MN}{LM}$$

Multiple manifestation

Now, $\frac{LM}{NL} \div \frac{MN}{NL} = \frac{LM}{MN} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

Again, $\frac{MN}{NL} \div \frac{LM}{NL} = \frac{MN}{LM} = \frac{\cos\theta}{\sin\theta} = \cot\theta$





Suppose, the angle adjacent to base of a right angled triangle is θ. In respect of angle θ we have learn six trigonometric ratios. They are: sinθ, cosθ, tanθ, cosecθ, secθ and cot θ

If we know the value of any one ratio, we can find the value of the other 5 ratios. For example,

If sin 30º= ½, let us find the value of cos 30º, tan 30º, cosec 30º, sec 30º and cot 30º.

Math Lab Activity

Topic : **Finding the value of trigonometric ratio**

Previous knowledge

- Concept of perpendicular, base and hypotenuse in respect of the angle which is not the right angle of the right angled triangle
- Concept of expressing the trigonometric ratio of the angle in respect of the length of the side

Learning objectives

- To apply trigonometric ratio for measuring the length and height of different objects and the value of different angles

Materials required

equipments of geometry box, white paper and a calculator

Execution of task

(i) Students will draw a right angled triangle with a set square.

(ii) They will measure and write the length of the sides of the triangle with a scale.

(iii) They will measure and write the angle with a protractor which is not a right angle.

(iv) They will write two trigonometric ratios (e.g. sin θ and tan θ) of the angle and fill up the following table.

(v) They will know the value of the trigonometric ratios from the teacher or by using a calculator and write them in the given table.

(vi) They will verify whether the ratio of the length of the sides is equal to the trigonometric ratio of the angle.

(vii) They will repeat the process by changing the length of the sides but keeping the value of the angle constant at θ.

Sl no.	Length of the perpendicular	Length of the base	Length of the hypotenuse	$\frac{\text{length of the perpendicular}}{\text{length of the hypotenuse}}$	$\frac{\text{length of the perpendicular}}{\text{length of the base}}$	θ	sin θ	tan θ

Math Lab Activity

Topic Applying trigonometric ratio to find the height of an object

Previous knowledge

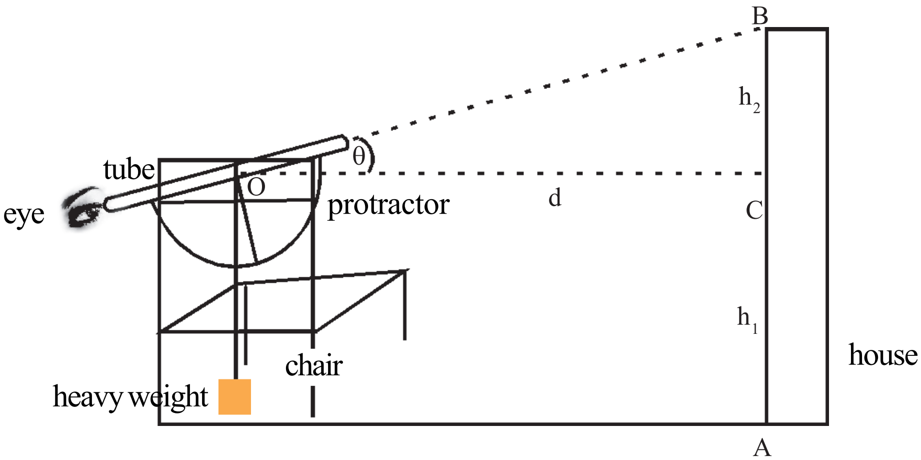
- To identify the perpendicular, base and hypotenuse of a right angled triangle in respect of a particular angle ı
- To know how to find the value of the trigonometric ratio of an angle

Learning objectives

- To find the height and distance of an object by applying trigonometric ratio

Materials required

- Clinometers (equipment to measure angle), measuring tape, white page, equipments of a geometry box



Execution of task

Making of a clinometer: A wooden protractor is fixed at the centre with a screw to a wooden chair (as shown in the diagram) in such a way that the protractor can rotate at the centre. An empty plastic pipe is glued to the protractor along its straight edge. A mass is hung with a string passed through the centre so that the protractor can remain static at an angle. Looking through the pipe from the bottom region one can observe the highest point of a high object at the opposite end and also measure its angle. The clinometer can also be manufactured in other ways.

(i) A clinometer has to be kept at the proper place to measure the height of a single storied house (AB).

(ii) The highest point of the house has to be observed by turning the protractor and looking through the pipe as shown in the diagram.

(iii) The angle created by the viewing line with the horizontal line passing through the centre of the protractor can be measured (θ).

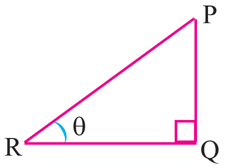
(iv) The distance 'd' (OC) of the house has to be measured by a measuring tape.

(v) The height 'h_1' (AC) from the base to the centre of the protractor has to be measured.

(vi) In the diagram, suppose BC= h_2

(vii) By placing the clinometer at different positions different measurements of θ, d and h_2 can be recorded.

(viii) Using the measurement of angle θ a right-angled triangle has to be drawn and the trigonometric ratio ($\tan\theta$) will be calculated by the length of its side. (ΔPQR, $\angle R=\theta$, $\angle Q = 90^{\circ}$)



(ix) The height (AB) of the house can be found by the value of $\tan\theta$ and the values of d, h_1 and h_2.

Observation:

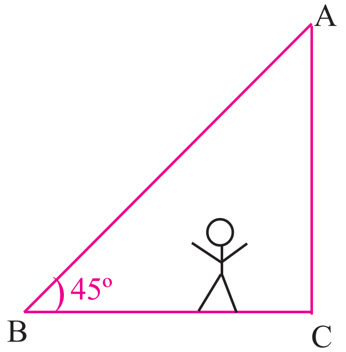
Serial no.	Value of θ	Value of $\tan\theta$	Value of d	Value of h_2 from $\tan\theta = \frac{h_2}{d}$	Value of h_1	Value of AB

Understanding why the trigonometric ratios of some particular angles are often discussed:

Sometimes we need to measure the height or distance of some things. But they are so high or distant that they cannot be measured by a measuring tape. So, we use trigonometric ratio.

(i) Suppose on a tour, I was curious to know the height of a tall tree. So from the base of the tree I walked away 26 steps at a certain direction. The place where I stood, I looked at the highest point of the tree. I found that line of my vision is making an angle of 45° with the base. Besides, in every step I had covered 1 meter distance.

From the above discussion I get a right angled triangle and thereby understand that the height of the tree is 26 meter.



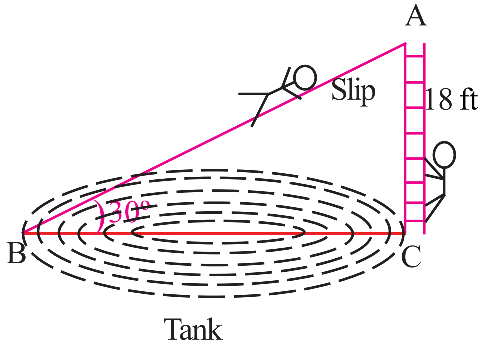
$$\tan 45^\circ = \frac{AC}{BC}$$

$$\therefore \quad 1 = \frac{AC}{BC}$$

$$\therefore \quad BC = AC$$

$$\therefore \quad AC = 26 \text{ meter } (\because BC = 26 \text{ meter})$$

(ii) A slip was installed in the playground infront of our house. Interestingly, the slip was installed over a small pool. There is an iron ladder on one side of the pool. Climbing up the ladder one can cross over the pool by the sliding on the slip. How can we measure the length of the slip? We found that the slip is making an angle of 30^0 with the ground. And the difference between the adjacent two horizontal rods of the ladder is 1 ft. The ladder is 18 ft. high. So I realized how length of the slip.



$$\sin 30^\circ = \frac{AC}{AB}$$

$$\therefore \quad \frac{1}{2} = \frac{AC}{AB}$$

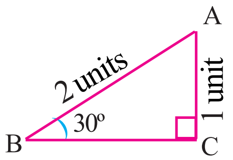
$$\therefore \quad AB = 2AC$$

$$\therefore \quad AB = 2 \times 18 \text{ ft.} = 36 \text{ ft.}$$

In a right angled triangle if we know the length of a side, measurement of an angle other than the right angle, then we can calculate the length of the other side using trigonometric ratio. However, we have seen that the known angles are always 30°, 45° or 60°. Why isn't the angles are of 10°, 20°, 40° or 50°?

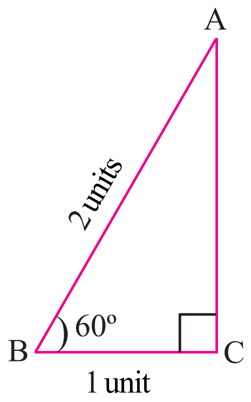
If the angle adjacent to the base of a right angled triangle is 30°, 45° or 60° then the triangle becomes quite interesting. Let's see what is it?

If the angle adjacent to the base is 30°, we measure the perpendicular and the hypotenuse and find that the length of the hypotenuse is double the length of the perpendicular.



i.e. If $AC = 1$ unit

$AB = 2$ units

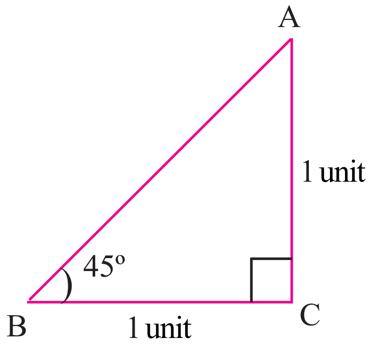


Again, in the triangle where the angle adjacent to the base is 60º by measuring the base and the hypotenuse, we will find that the length of the hypotenuse is double the length of the base.

i.e. If BC = 1 unit

AB = 2 units

In the triangle where the angle adjacent to the base is 45º by measuring the base and the perpendicular, we will find that the length of the perpendicular and the base are equal.



If the angle adjacent to base of a right angled triangle is 10º, 20º, 40º etc. then the length of a side will not be double, three times, four times or five times the length of the other side.

Hence, in a right angled triangle the ratio of the two sides or trigonometric ratio is ½ or 1 etc. like normal number if the angle adjacent to the base is 30º, 45º or 60º. So we generally work on these three angles.

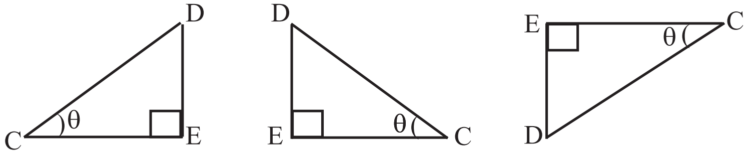
There are two interesting right angled triangles where the angles adjacent to the base are 0º and 90º. The triangle is queer. By drawing the triangle we get,

See page nos. 297 and 298 of Ganit Prakash for class X

worksheet : 1

(A) $\angle DEC = 90^\circ$ and $\angle DCE = \theta$ of right angled triangle CDE, fill in the blanks:

(i)



In the diagram, even if right angled triangle CDE is turned in different direction, the length of the base and hypotenuse remains ________________

(ii) The trigonometric ratios in respect of angle θ are

(a) $\dfrac{\text{Length of DE}}{\text{Length of CD}} = \dfrac{\text{Length of perpendicular}}{\text{Length of hypotenuse}} =$ ______________

(b) $\dfrac{\text{Length of CE}}{\text{Length of CD}} = \dfrac{\text{Length of base}}{\text{Length of hypotenuse}} =$ ______________

(c) $\dfrac{\text{Length of CD}}{\text{Length of DE}} = \dfrac{\text{Length of hypotenuse}}{\text{Length of perpendicular}} =$ ______________

(d) $\dfrac{\text{Length of CD}}{\text{Length of CE}} = \dfrac{\text{Length of hypotenuse}}{\text{Length of base}} =$ ______________

(e) $\dfrac{\text{Length of CE}}{\text{Length of DE}} = \dfrac{\text{Length of base}}{\text{Length of perpendicular}} =$ ______________

(f) $\dfrac{\text{Length of DE}}{\text{Length of CE}} = \dfrac{\text{Length of perpendicular}}{\text{Length of base}} =$ ______________

(iii) If $\theta = 30^\circ$, CD = 2 unit and DE = 1 unit then ______________

(a) $\sin 30^\circ =$ ____ (b) $\cos 60^\circ =$ ____

(iv) If DE and CE are equal in length, then the value of $\tan 45^\circ$ is ______________

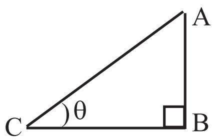
(B) Choose the correct answer :

(i) ABC is a right angled triangle where $\angle ABC = 90^\circ$, $\angle BCA = \alpha$ and $\angle CAB = \theta$, the perpendicular with respect to angle θ and the base with respect to angle α is/are

(a) AB (b) BC (c) AC (d) AB ও BC

(ii) In a right angled triangle ABC, if $\angle ABC = 90^\circ$, $\angle BCA = \theta$, $\tan\theta = \frac{2}{3}$ and AB = 6 cm, then the length of BC is

(a) $\frac{1}{6}$ cm. (b) 3 cm. (c) 9 cm. (d) 4 cm.



(iii) The correct relation is

(a) $\sin\theta \cos\theta = 1$ (b) $\sec\theta \operatorname{cosec}\theta = 1$ (c) $\operatorname{cosec}\theta \cot\theta = 1$ (d) $\sin\theta \operatorname{cosec}\theta = 1$

(iv) $\frac{\sin 10^\circ}{\cos 10^\circ} =$

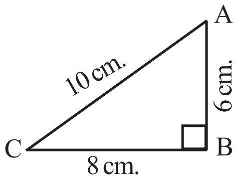
(a) $\cot 10^\circ$ (b) $\tan 20^\circ$ (c) $\tan 10^\circ$ (d) $\tan 1^\circ$

(v) The value of each member of the pair not grater than 1 is

(a) $\sec\theta, \sin\theta$ (b) $\operatorname{cosec}\theta, \cos\theta$ (c) $\sec\theta, \cos\theta$ (d) $\sin\theta, \cos\theta$

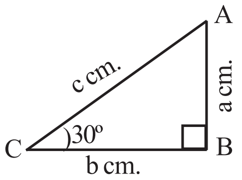
(C) Write True/False:

(i)



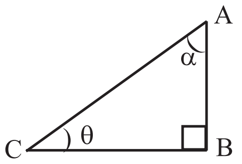
In the diagram, $\sin A = \frac{3}{5}$

(ii)



In the diagram, $\cos 30^\circ = \frac{a}{c}$

(iii)



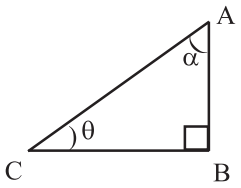
In the diagram, $\tan\theta \times \tan\alpha = 1$

(iv) If the value of tangent of an angle other than the right angle of a right angled triangle be 1 then the triangle will be a right angled isosceles triangle.

D. Answer the following questions:

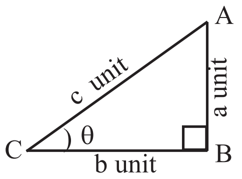
(i) In a right angled triangle the value of which trigonometric ratio is not greater than 1 and why?

(ii)

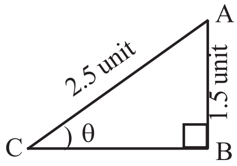


In the diagram, if $\sin\theta = \cos\theta$ then find the value of α

(iii)



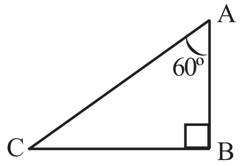
From the diagram, find the value of $\sin^2\theta + \cos^2\theta$

(iv) 

From the diagram, find the value of $\sin\theta + \cos\theta$

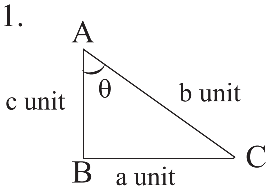
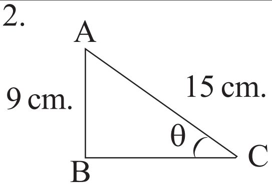
E. Answer the following questions:

(i) In a right angled triangle if θ is a positive acute angle and, $\sin\theta = \frac{2}{3}$ then find the value of $\cot\theta$

(ii) 

In the diagram if $\cos 60^\circ = \frac{1}{2}$ then find the value of $\cot 30^\circ$

Worksheets and task analysis :

Task)	Student's Responses	Teacher's Reflections
1.  In the diagram, in ΔABC $\sin\theta = ?$	(i) $\sin\theta = \frac{\text{Length of perpendicular}}{\text{Length of hypotenuse}} = \frac{c}{b}$ (ii) $\sin\theta = \frac{\text{Length of perpendicular}}{\text{Length of hypotenuse}} = \frac{c}{a}$ (iii) $\sin\theta = \frac{\text{Length of perpendicular}}{\text{Length of base}} = \frac{a}{c}$ (iv) $\sin\theta = \frac{\text{Length of perpendicular}}{\text{Length of hypotenuse}} = \frac{a}{b}$	(i) The student has learnt trigonometric ratio but could not recognize perpendicular and base with respect to the angle θ. (ii) S/he has learnt trigonometric ratio but could not recognize the perpendicular with respect to the angle θ and hypotenuse. (iii) S/he could recognize perpendicular and base in respect to angle θ but could not understand trigonometric ratio. (iv) S/he could recognize the perpendicular with respect to angle θ and the hypotenuse with respect to the right angle and also understood trigonometric ratio of angle.
2.  In the diagram, in Δ ABC what is the value of $\cos\theta$?	(i) $\cos\theta = \frac{\text{Length of base}}{\text{Length of hypotenuse}} = \frac{9}{15} = \frac{3}{5}$ (ii) $\cos\theta = \frac{\text{Length of perpendicular}}{\text{Length of hypotenuse}} = \frac{9}{15}$ (iii) $AC^2 = AB^2 + BC^2 \Rightarrow 15^2 = 9^2 + BC^2$ $\Rightarrow BC^2 = 15^2 - 9^2 = 144 \therefore BC = 12$ $\cos\theta = \frac{\text{Length of base}}{\text{Length of hypotenuse}} = \frac{12}{15} = \frac{4}{5}$	(i) S/he has learnt to express cos θ with respect to the respective side but is unable to identify the angle θ with respect to the base. (ii) S/he cannot express cosθ with respect to the length of the side, but can identify the respective sides with respect to the angle θ. S/he is unable to understand the application of Pythagoras theorem. (iii) S/he can identify the base and the perpendicular with respect to the angle θ. S/he also understands the application of Pythagoras theorem. S/he even understands the trigonometric ratio of angles.

Sample of some probable misconceptions of trigonometric ratio and remedials

Probable Misconceptions	Remedial
• $\tan\theta = 1,\ \theta = ?$ $\therefore\ \theta = \dfrac{1}{\tan}$	• $\tan\theta = 1,$ $\therefore\ \theta = 45^{\circ}$ [$\tan\theta = \dfrac{\text{length of the perpendicular}}{\text{length of the base}} = 1$ $\therefore$ length of the perpendicular = length of the base $\therefore$ The triangle is an isosceles right angled triangle $\therefore\ \theta = 45^{\circ}$]
• $\tan\theta = \dfrac{3}{4}, \cos\theta = \dfrac{4}{5}$ $\sin\theta = ?$ $\tan\theta = \dfrac{3}{4}$ Or, $\dfrac{\sin}{\cos} = \dfrac{3}{4}$ Or, $\sin = \dfrac{3}{4}\cos$ $= \dfrac{3}{4}\times\dfrac{4}{5} = \dfrac{3}{5}$ $\therefore\ \sin\theta = \dfrac{3}{5}$	• $\tan\theta = \dfrac{3}{4}$ Or, $\dfrac{\sin\theta}{\cos\theta} = \dfrac{3}{4}$ Or, $\sin\theta = \dfrac{3}{4}\times\cos\theta$ $= \dfrac{3}{4}\times\dfrac{4}{5} = \dfrac{3}{5}$ $\sin\theta = \dfrac{3}{5}$
• (i) If $\cos\theta = \dfrac{4}{5}$ $\dfrac{\sin\theta + \cos\theta}{\sin\theta} = ?$ $\dfrac{\cancel{\sin\theta} + \cos\theta}{\cancel{\sin\theta}} = \cos\theta = \dfrac{4}{5}$ (ii) $\cos\theta = \dfrac{4}{5} = \dfrac{\text{length of base}}{\text{length of hypotenuse}}$ Lenght of base = 4 or, base = 4 Length of hypotenuse = 5 or, hypotenuse = 5	• $\cos\theta = \dfrac{4}{5} = \dfrac{\text{length of the base}}{\text{length of the hypotenuse}}$ Suppose the length of the base = 4k unit $\therefore$ The length of the hypotenuse = 5k unit To get the length from the ratio, a common factor k (k>0) has to be multiplied. From Pythagoras theorem we get, (length of the base)^2 + (length of the perpendicular)^2 = (length of the hypotenuse)^2 Or, $16k^2$ + (length of the perpendicular)$^2 = 25k^2$ Or, (length of the perpendicular) $= \sqrt{25k^2 - 16k^2}$ $= 3k$ (length is always positive) $\therefore \sin\theta = \dfrac{3k}{5k} = \dfrac{3}{5}$ $\therefore \dfrac{\sin\theta + \cos\theta}{\sin\theta} = \dfrac{\frac{3}{5}+\frac{4}{5}}{\frac{3}{5}} = \dfrac{7}{5}\times\dfrac{5}{3} = \dfrac{7}{3}$ [It can also be done by another process]

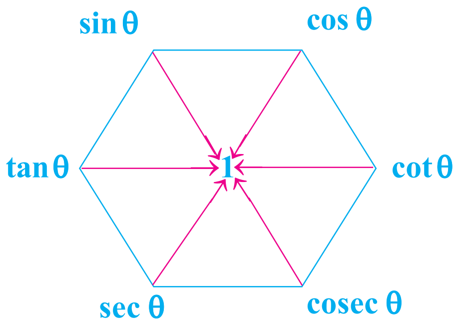
Probable Misconceptions	Remedial
• $\sin^2\theta = \sin\theta^2$	• $\sin^2\theta = \sin\theta\times\sin\theta = (\sin\theta)^2$ $\sin\theta^2 = \sin(\theta\times\theta)$ $\therefore$ $\sin^2\theta \neq \sin\theta^2$
•	• $\sin 2\alpha \neq 2\sin\alpha$ [e.g. $\sin 60^\circ = \frac{\sqrt{3}}{2}$ $2\sin 30^\circ = 2\times\frac{1}{2} = 1$]
• $\frac{\cancel{\sin}\,\theta}{\cancel{\sin}\,\alpha} = \frac{\theta}{\alpha}$	• $\frac{\sin\theta}{\sin\alpha}$ $\sin\theta$ is not product of sin and θ $\sin\alpha$ is not product of sin and α $\frac{\theta}{\alpha} \rightarrow$ the quotient of the measurements of two angles Here the two 'sin' can not be cancelled
• $\sin A \pm \sin B = sin(A \pm B)$	• sin A and sin B, each is the ratio of the lengths of two sides i.e. not sin × A or sin × B

Learning Outcomes :

The learners will be able to —

- identify the base and hypotenuse of a right angled triangle with respect to an angle other than the right angle.
- express the trigonometric ratios with respect to an angle of a right angled triangle other than the right angle.
- find the height and distance of an object using the trigonometric ratio.
- use the trigonometric ratio in solving real life problems.

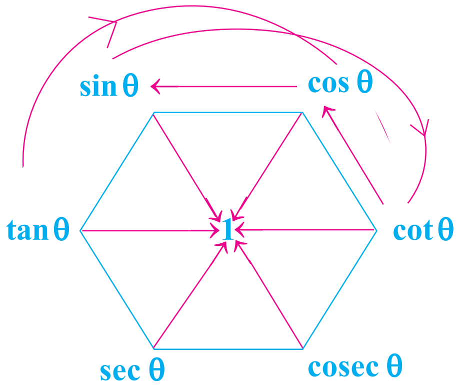
We find some trigonometric identities joyfully :



$\sin\theta \times \operatorname{cosec}\theta = 1$

$\cos\theta \times \sec\theta = 1$

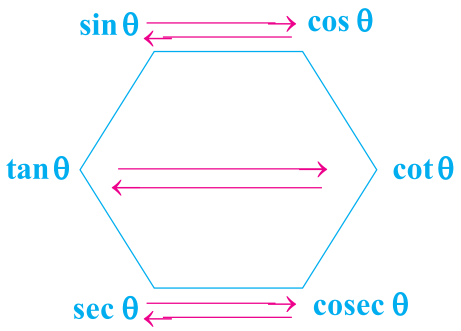
etc.



$\tan\theta \times \cos\theta = \sin\theta$

$\sin\theta \times \cot\theta = \cos\theta$

etc.



$\sin(90^\circ - \theta) = \cos\theta$

$\tan(90^\circ - \theta) = \cot\theta$

etc.

In this way the students will think about some more trigonometric rules with a hexagon.

Statistics (Ogive)

Activity

(Topic) **Drawing an ogive (less than type)**

(Previous Knowledge)
- Concept of graph
- Concept of data, freequency, continuous ans discontinuous class interval
- Concept of cumulative frequency distribution

(Learning objectives)
- To know the data at a glance
- To understand easily the analysis of the data
- To develop some idea about probability of some aspects of the data

(Material required) art paper, graph paper, equipments of the geometry box, colour pencil, glue, cello tape and a pair of scissors

(Execution of task) :

(Observation): A shopkeeper wants to store some garments of different prices. He prepares a list of the garments sold at different price in the previous month to get an idea of the stock to be made.

(Contextualization) :

Price of garments (in Rupees)	**No. of garments**
0-300	10
300-600	15
600-900	18
900-1200	30
1200-1500	40
1500-1800	35
1800-2100	12
2100-2400	05

(i) The students will collect data like this and prepare a frequency distribution table of 8 classes on basis of the data.

(Cognitive Apprenticeship and Collaboration) :

The students will do the following task in groups. They will discuss among themselves about the following tables. The teacher will help them if required.

Price of garments (in Rupees)	No. of garments
Less than 0	0
x_1 — x_2 (0) (300)	f_1 (10)
x_2 — x_3 (300) (600)	f_2 (15)
x_3 — x_4 (600) (900)	f_3 (18)
x_4 — x_5 (900) (1200)	f_4 (30)
x_5 — x_6 (1200) (1500)	f_5 (40)
x_6 — x_7 (1500) (1800)	f_6 (35)
x_7 — x_8 (1800) (2100)	f_7 (12)
x_8 — x_9 (2100) (2400)	f_8 (5)

The students will make a cumulative frequency distribution table(less than type) from the data given above :

Price of garments (in rupees)	**Cummulative frequency (less then type)**
Less than x_1 (0)	$f_0 = (0) = y_0$
Less than x_2 (300)	Suppose $f_1 = (10) = y_1$
Less than x_3 (600)	$f_1+f_2=(25)=y_2$
Less than x_4 (900)	$f_1+f_2+f_3=(43)=y_3$
Less than x_5 (1200)	$f_1+f_2+f_3+f_4=(73)=y_4$
Less than x_6 (1500)	$f_1+f_2+f_3+f_4+f_5=(113)=y_5$
Less than x_7 (1800)	$f_1+f_2+f_3+f_4+f_5+f_6=(148)=y_6$
Less than x_8 (2100)	$f_1+f_2+f_3+f_4+f_5+f_6+f_7=(160)=y_7$
Less than x_9 (2400)	$f_1+f_2+f_3+f_4+f_5+f_6+f_7+f_8=(165)=y_8$

Interpretation Construction :

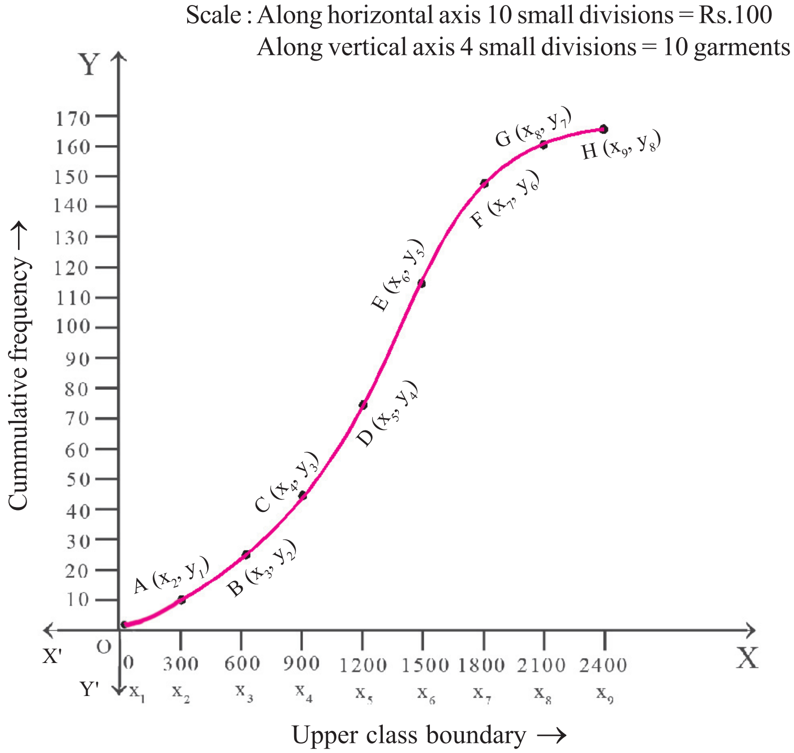
(iii) A graph paper was glued to an art paper.

(iv) Two straight lines XOX' and YOY' intersect each other perpendicularly at the point O in the graph paper. A convenient scale is choosen.

(v) The points O (x_1, y_0), A (x_2, y_1), B (x_3, y_2), C (x_4, y_3), D (x_5, y_4), E (x_6, y_5), F (x_7, y_6), G (x_8, y_7), H (x_9, y_8), are to be marked in the graph paper.

- The scale will be formed on the vertical and horizontal axes.
- The upper limits of the class intervals are to be marked on the horizontal axis and and their corresponding cummulative frequencies are shown on the vertical axis.
- The points will be marked where x-coordinates is the upper class boundary and y- coordinates is the cumulative frequency of the class.
- The points are to be joined from left to right.
- The points are to be joined with a coloured pencil without using a scale.

In the diagram it is shown —



The class intervals are :

$x_0 - x_1 = 0$, $x_1 - x_2 = 0 - 300$, $x_2 - x_3 = 300 - 600$, ..., $x_8 - x_9 = 2100 - 2400$

Here $f_1 = 10$, $f_2 = 15$, ..., $f_8 = 05$

$f_1 = y_1 = 10$, $y_2 = 25$, ..., $y_8 = 165$

The coordinates of A= (300, 10), coordinates of B = (600, 25)............ coordinate of H = (2400, 165).

By joining the points A, B, C, D, E, F, G and H are joined without a scale. The graph that is found is the cumulative frequency curve (less than type) or Ogive.

The cumulative frequency curve is increasing from less to greater frequency. So the curve is moving upwards. This is the less than type Ogive.

(i) The number of garments sold below a certain price can be found from the above data.

(ii) The maximum number of garments sold at which price can be found from the above data.

But (iii) from which graph shall we get the number of garments sold in excess of a certain fixed price? To know about it, does some part of the less than type Ogive need to be modified?

(iv) What will be the point of intersection of the less than type Ogive and the greater than type Ogive?

(v) What measurement of central tendency can be found from the intersection of the two Ogives?

(vi) What is the approximate measurement of the central tendency?

Multiple Interpretation : The learners will use the data to draw the Ogives along the x-axis and y-axis based on the different scales.

Multiple Manifestation : In case of varied data, one can gain knowledge by forming Ogives.

Based on these activities some probable errors or misconceptions and their remedials are given below:

<table>
<tr><th>Probable errors</th><th>Remedial</th></tr>
<tr><td>• To draw the less than type Ogive students plot the upper class limit along the horizontal axis and their corresponding frequency along verticle axis.</td><td>• To draw less than type Ogive students plot the upper class limit along the horizontal axis and their corresponding cumulative frequencies along the vertical axis.</td></tr>
<tr><td>• They are unable to identify median class of the cumulative frequency distribution table i.e. they do not know how to calculate the median.</td><td>• To find out the median class the cumulative frequency is to be calculated which is half or little greater than half of the total frequency in the respective class.</td></tr>
<tr><td>• They are unable to find the value of the symbol used in the formula from the table for calculating the median of the given data (for example : place the cumulative frequency of the median class in place of the cumalative frequency just before median class</td><td>• The proper value of the symbol will be placed in the formula.
$$M = l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times h$$</td></tr>
<tr><td>• In Cumulative frequency, if the frequency of a class interval is x then by writing 60x inplace of (60+x) as the cumulative frequency of that class will be erroneous.</td><td>• $\because 60 + x \neq 60x$</td></tr>
<tr><td>• At the time of selection of class interval even though the lower class interval is at the origin, use kink while drawing the Ogive.</td><td>• Kink should not be used when the lower class interval is at the origin while drawing the Ogive.</td></tr>
<tr><td>• The students are not sure about the continuity of the class intervals and will use the fragmented class intervals of the frequency distribution table.
<table>
<tr><td>Marks</td><td>0-4</td><td>5-9</td><td>10-14</td><td>15-19</td><td>20-24</td></tr>
<tr><td>No. of girl Students</td><td>4</td><td>5</td><td>7</td><td>8</td><td>7</td></tr>
</table></td><td>• The students should make sure about the continuity of the class intervals and will use the continuous table.
<table>
<tr><td>Marks</td><td>0.5-4.5</td><td>4.5-9.5</td><td>9.5-14.5</td><td>14.5-19.5</td><td>19.5-24.5</td></tr>
<tr><td>No.of girl Students</td><td>4</td><td>5</td><td>7</td><td>8</td><td>7</td></tr>
</table></td></tr>
<tr><td>• The value of the ordinate of the intersecting point of the less than type ogive and greater than ogive is to be considered as the value of the median.</td><td>• The value of the abscissa of the intersecting point of the less than type ogive and greater than ogive is to be considered as the value of the median.</td></tr>
<tr><td>• The scale is not shown in the graph while drawing the ogive.</td><td>• The scale must be shown in the graph while drawing the ogive.</td></tr>
</table>

Worksheet :

1. Choose the correct answer :

(i) The central tendency which is measured by the abscissa of the intersecting point of the less than type Ogive and greater than type Ogive curves is represented by

(a) mean (b) median (c) mode (d) mean and mode

(ii)

Class Interval	10-20	20-30	30-40	40-50	50-60
Frequency	7	9	13	7	8

The median class of the above frequency distribution is

(a) 50–60 (b) 40–50 (c) 30–40 (d) 20–30

(iii) The value of the abscissa of the points to draw the less than type ogive is

(a) lower class boundary of the class limits (b) upper class boundary of the class limits

(c) class mark (d) cumulative frequency of the classes

(iv) If the data in the ascending order is 10, 20, 30, x+1, x+3, 60, 70, 80 and the median of the data is 50, then the value of x is

(a) 40 (b) 50 (c) 48 (d) 52

2. Write True/False :

(i) If the data 10, 13, 32, 43, a, b, 60, 65, 72 is arranged in the ascending order, then the median is a+b/2

(ii) The median of the data 1, 2, 5, 3, 6, 4 is 3.

(iii) The lower class boundary of each class, is used to plot the points for drawing less than type Ogive.

(iv) The value of the median can be determined from either the less than type ogive or the greater type ogive.

3. Fill in the blanks :

(i) To draw an ogive the class boundaries are______________.

(ii) The graph of the cumulative frequency is known as ______________.

(iii) _______ of the total frequency lies below the point o intersection between the two ogives of less than type and greater than type of the same data.

(iv) ___________ freequency represents along the vertical axis for drawing the ogive.

4. Answer the following questions :

(i) The median of 20, 14, 15, 17, 22, and 2

(ii)

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	10	7	9	13	7	8

From the above table find the cumulative frequency of 20 or excess than 20 of the greater than type.

(iii) Make a less than type cumulative frequency distribution table of the data given in question 4(ii).

(iv) Find the cumulative frequency of 40 or less than 40 of the less than type ogive from the given data.

5. Draw the cumulative frequency curve of the greater than type based on the data given in Question 4 (ii).

Expected Learning Outcomes :

The learners will be able to —

- enlist data into a tabular form
- transfer the raw data into arranged data
- transfer continuous distribution table from the discontinuous distribution table
- draw different graphs (from less than type ogive to greater than type ogive) of the data
- interpret the data by the ogive curve
- infer by analyzing the data

Internal Formative Evaluation
Guidelines for Implementation

1. Survey

Selected text : Linear quadratic equation with one variable (chapter 1)

Step I: The teacher will instruct the students to read the chapter -1 carefully. Every student will have to set a question paper individually on the mentioned chapter. The questions should be set on different sections of this chapter i.e. all the 5 questions should not be of the same type. Each paper setter will write his/her name and roll number on his/her question paper.

A sample question paper is given below:

(i) The total amount will be Rs.35 if each of x students contributes Rs. x and our teacher contributes Rs. 10. Express the above statement in the form of linear quadratic equation with one variable.

(ii) Form a quadratic equation whose two roots are 3 and 4

(iii) Solve the equation $x^2 - 7x + 10 = 0$

(iv) If the roots of the equation $2x^2 + 3x + 5 = 0$ be m and n, then find the value of m + n and mn.

(v) For what condition of a, b, c the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$will be real and equal ?

Step II: According to the instruction of teacher each student will form an answer script of his/her question paper by any other student.

Step III: The paper setter will evaluate the answer script of his/her question paper. S/he will give (✓) mark for correct answer and (×) mark for wrong answer.

Step IV: All the students will submit the question papers set by them and the answer scripts of those question papers to the teacher at the same time .

Step V: The teacher will award marks to the paper setter after assessing his/her question paper and answer script.

Allotment of marks :

Paper setting : 5 marks

Evaluating the answers correctly : 5 marks

Time allotted : 2 periods.

2. Nature Study

Selected text: Right circular cylinder (Chapter 8)

Step I: The teacher will tell the students to think about the right circular cylinder shaped objects which can be seen in environment i.e. around their surrounding where they live. After that the teacher will instruct the students to read the chapter carefully.

Step II: The teacher will write down some questions on the black board

A sample question paper is given below :

(i) Write four right circular cylinder shaped objects seen in environment.

(ii) Draw the pictures of any two of the above named objects.

(iii) Find the volume and total surface area of any one drawn picture.

Step III: Every student will have to prepare his/her answer script and submit it to the teacher after writing his/her name and roll number. The teacher will award marks after assessing his/her answer script.

Allotment of marks:

2 Marks for number (i) of the sample question paper

3 Marks for number (ii) of the sample question paper

5 Marks for number (iii) of the sample question paper.

Time allotted : 2 periods

3. Case study

Selected text : Quadratic surd (Chapter 9)

Step I: The teacher will instruct the students to read the chapter carefully.

Step II: The teacher will write down same questions on the black board on the concept of quadratic surd.

A sample question paper is given below:

(i) If $x^2 = 5$ then what is the value of x ? Are two values of x rational numbers or irrational numbers?

(ii) We know that $\sqrt{a^2} = |a|$. What is the value of $\sqrt{(-5)^2}$?

(iii) Add: $(3\sqrt{7} + 3\sqrt{2}) + (4\sqrt{7} - 3\sqrt{2})$

(iv) Write a number by which $(2 + 3\sqrt{2})$ be multiplied so that the product will be a rational number.

(v) What will be the quotient when $(\sqrt{12} + \sqrt{48})$ is divided by $\sqrt{3}$? Is the quotient a rational number or irrational number?

Step III: Every student will have prepare his/ her answer script and submit it to the teacher after writing his/ her name and roll number. The teacher will award marks to each student after assessing his/her answer script.

Allotment of marks:

2 marks for each question.

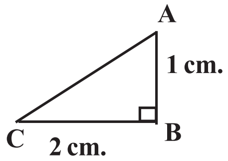
Time allotted: 2 periods.

4. Creative Writing

Selected text: Pythagoras Theorem (Chapter 22):

Step I: The teacher will write the question given below on the black board and discuss with the students.

We have seen that in a right angled triangle if the lengths of the sides containing the right angle are rational numbers the length of the hypotenuse may be an irrational number.



For example, the figure given at the right side, ABC is right angled triangle and the lengths of the sides containing the right angle are 1 cm. and 2 cm.

But the length of the hypotenuse $AC = \sqrt{1^2 + 2^2}$ cm. $= \sqrt{1+4}$ cm $= \sqrt{5}$ cm., which is an irrational number.

Step II: The teacher will instruct the students to write examples of three right angled triangles of which

(i) The lengths of the sides containing the right angle are rational numbers and the length of the hypotenuse is also a rational number.

(ii) The lengths of the sides containing the right angle are irrational numbers and the length of the hypotenuse is a rational number.

(iii) The length of one side containing the right angle is a rational number but the length of the other side is an irrational number and the length of the hypotenuse is a rational number.

The teacher will write the above statements on the black board and instuct the students to draw rough sketches of the triangles on their answer scripts.

Step III: Every student will have prepare his/her answer script and submit it to the teacher after writing his/her name and roll number. The teacher will award marks to each student after assessing his/her answer script.

Allotment of marks:

3 marks for question number (i)

3 marks for question number (ii)

4 marks for question number (iii)

Time allotted : 2 periods.

5. Model making

Selected text: Cuboid (Chapter 4):

Step I: The teacher will instruct each student to make a cuboid in the class room by cutting a piece of card board. (There should be arrangenment of cellotapes, scissors, card boards etc.) The teacher will also instruct the students to measure the length, breadth and height of the cuboid made by him by the ruler. Then each of them will write the length, breadth, height, total surface area and volume on the surface of the cuboid or in a separate paper.

Step II: The teacher will award marks to each student for the constuction of the model correctly finding the total surface area and volume.

Allotment of marks:

(i) 5 marks for construction of cuboid

(ii) 2 marks for finding volume. 1 mark will be deducted for wrong/ without unit.

(iii) 3 marks for finding total surface area. 1 mark will be deducted for wrong/ without unit.

Time allotted : 2 periods.

6. Open textbook Evaluation (OTBE)

Selected text: Linear quadratic equation with one variable (Chapter 1):

Step I: The teacher will solve two equations on the black board.

For example:

(1) $x^2 + 4\sqrt{2}x + 6 = 0$

$\Rightarrow x^2 + \sqrt{2}x + 3\sqrt{2}x + 6 = 0$

$\Rightarrow x(x + \sqrt{2}) + 3\sqrt{2}(x + \sqrt{2}) = 0$

$\Rightarrow (x + \sqrt{2})(x + 3\sqrt{2}) = 0$

$\Rightarrow x + \sqrt{2} = 0 \quad or \quad x + 3\sqrt{2} = 0$

$\Rightarrow x = -\sqrt{2}x \quad or \quad x = -3\sqrt{2}$

Two roots of the equation are $-\sqrt{2}$ and $-3\sqrt{2}$

2) $x^4 - 17x^2 + 16 = 0$

$\Rightarrow$ $y^2 - 17y + 16 = 0$ (Let, $x^2 = y$.)

$\Rightarrow$ $y^2 - 16y - y + 16 = 0$

$\Rightarrow$ $y(y-16) - 1(y-16) = 0$

$\Rightarrow$ $(y-16)(y-1) = 0$

$\Rightarrow$ $y - 16 = 0$ or $y - 1 = 0$

$\therefore$ $y = 16$ or $y = 1$

$\therefore$ $x^2 = 16$ or $x^2 = 1$ $(\because y = x^2)$

$\therefore$ $x = \pm\sqrt{16}$ or $x^2 = \pm\sqrt{1}$

$\therefore$ $x = \pm 4$ or $x = \pm 1$

$\therefore$ Four roots of the equation are $4, -4, 1, -1$

Step II: Ask each of the students carefully notice and understand the solutions of two equations. Give them time for understanding.

Step III: The teacher will give two equations, the solution of which will require the concept of the previous two equations. The problems should not be in the text book. For example,

i) $x^2 + 113\sqrt{3}x + 3636 = 0$

ii) $x^3 - 173x^2 + 676 = 0$

Step IV: The students will try to solve the two equations on their answer script.

Step V: The teacher will award marks to each student after assessing his/her answer script. Correctness in every step of the solution should be considered.

Allotement of marks:

5 marks for question number (i)

5 marks for question number (ii)

(The students can take the help of their text book in the class)

Time allotted : 2 periods.

Note: Some examples of Internal Formative Evaluation on six different areas are given here with reference to certain chapters of the mathematics text book *Ganit Prakash* for Class X. In this way the teachers will assess the learners in reference to *Ganit Prakash* in a much better way.

Marks distribution of First Summative Evaluation

(Summative-I)

Subject	MCQ	SA	LA**	Total Marks
Arithmetic	2 (1×2)	2 (2×1)	5 (5×1)	9
Algebra	2 (1×2)	2 (2×1)	10 (3+4+3)	14
Geometry	2 (1×2)	4 (2×2)	5 (5×1)	11
Mensuration	-	2 (2×1)	4 (4×1)	6
Total Marks	**6**	**10**	**24**	**40**
	6 + 10 = 16			

Internal assessment : 10 Marks

** **L.A.**

Arithmetic

(i) Simple interest
(ii) Compound interest
(iii) Uniform rate of increase or decrease
— 1 out of 2 questions : 5×1 marks = 5 marks

Algebra

(i) Solution of Quadratic equation in one variable — 1 out of 2 questions : 3×1 marks = 3 marks

(ii) Application of quadratic equation in real problems [Construction of equation and solution] — 1 out of 2 questions : 4×1 marks = 4 marks

(iii) Ratio and proportion
(iv) Quadratic Surd
— 1 out of 2 questions : 3×1 marks= 3 marks

Geometry

(i) Theorem related to circle
(ii) Theorem related to angle on a circle
(iii) Theorems related to cyclic quadriateral
Theorem — 1 out of 2 questions : 5×1 marks = 5 marks

Mensuration

(i) Cuboid
(ii) Right circular cylinder
— 1 out of 2 questions : 4×1 marks = 4 marks

FIRST SUMMATIVE EVALUATION

MODEL QUESTION

TIME: 1 HOUR 30 MINUTES **FULL MARKS: 40**

1. Choose the correct answer : **1×6=6**

(i) A person gets an amount of Rs. 144 through compound interest by keeping Rs. 121 in a bank for 2 years. The annual rate of compound interest is

(a) $9\frac{1}{11}\%$ (b) 2% (c) 5% (d) 10%

(ii) If the annual rate of simple interest rises from 4% to 5% then the annual income of a person increases by Rs.1000. The capital of the person is

(a) 4000 (b) 5000 (c) 100000 (d) 50000

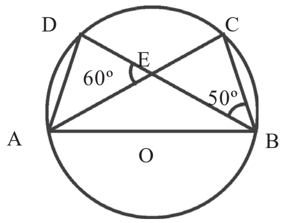
(iii) The quadratic equation having the roots -3 and 2 is

(a) $x^2+(-3+2)x+(-3)(2)=0$ (b) $x^2-(-3+2)x+(-3)(2)=0$

(c) $x^2-(-3+2)x-(-3)(2)=0$ (d) $x^2+(-3+2)x-(-3)(2)=0$

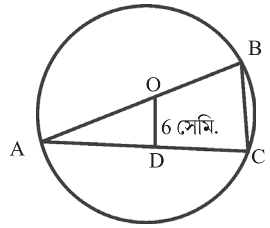
(iv) If the roots of the equation $x^2+x-1=0$ are α and β, then the value of $\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$ is

(a) -1 (b) 0 (c) 1 (d) ± 1



(v) If $\angle AED=60^0$ and $\angle CBE=50^0$ in a circle with centre O as shown in the given diagram, then the value of $\angle ADB$ is

(a) 60° (b) 110° (c) 70° (d) 90°



(vi) AC is a chord of a circle with centre O as shown in the given diagram. AB is the diameter. OD is a perpendicular on AC and if OD= 6cm, then the length of BC is

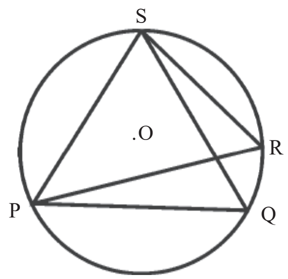
(a) 3 cm. (b) 6 cm. (c) 9 cm. (d) 12 cm.

2. Answer the following questions : **2×5=10**

(i) In how many years the principal will will be thrice if the rate of simple interest is 10% per annum?

(ii) If x: y=3: 5, then find the value of (4x + 3y) : (5x - y)

(iii) Two non-concentric circles of radius 5 cm each intersect each other. If the length of their common chord is 8 cm, find the distance between the centers of the two circles.



(iv) In the adjoining figure O is the centre of the circle. If $\angle QPS=70^\circ$ and $\angle QSR=40^\circ$, find the value of $\angle RQS$.

(a) 40° (b) 45° (c) 30° (d) 60°

(v) If all the sides of a cuboid are increased by 20%, what percent will the volume of the cuboid increase?

3. Find the compound interest of a sum of Rs. 50,000 at 10 % annual interest for 2 years?

(Or)

The value of a machine in a factory diminishes by 10% every year. After 3 years if the value of the machine is Rs. 43740, what is its present value? **5**

4. Solve: (any one) **$3\times1=3$**

(i) $\frac{1}{x} - \frac{1}{x+b} = \frac{1}{a} - \frac{1}{a+b}$, $x \neq 0, -b$

(ii) $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b$, $[x \neq \frac{1}{a}, \frac{1}{b}]$

5. The area of a rectangular field is 2000 sq. m. and its perimeter is 180. Find the length and breadth of the field.

(Or)

If the price of pens falls by Rs. 6 per dozen, then 3 more pens can be bought at Rs. 30. What is the price of the pens before it fell? **4**

6. If $\sqrt{a}$, $\sqrt{b}$ and $\sqrt{c}$ are in continued proportion than show that $(a+c)^2-b^2 = a^2+b^2+c^2$

(Or)

If $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $xy = 1$ than show that $\frac{x^2-xy+y^2}{x^2+xy+y^2} = \frac{61}{63}$ **3**

7. Prove that all angles subtended at the circle in the same segment are equal.

(Or)

Prove that if a perpendicular is drawn from the centre to the chord, which is not the diameter, the perpendicular will bisect the chord.

8. When75 buckets of water is taken from a cubical shaped cistern completely filled with water, $\frac{2}{3}$ water remains in the cistern. If the length of one side of the cistern is 1.5 m. how much water does each bucket contain?

(Or)

Water contains in a long gas-jar of 10 cm. radius. A piece of solid iron right circular cylindrical shaped of 8cm long and 5 cm diameter is fully immersed in that water. How much will the water level rise? **4**

Marks distribution of second summative Evaluation

(Summative-II)

Subject	MCQ	SA	LA**	Total Marks
Arithmetic	1 (1×1)	-	5 (5×1)	6
Algebra	2 (1×2)	2 (2×1)	3 (3×1)	7
Geometry	2 (1×2)	2 (2×1)	13 (5+5+3)	17
Mensuration	2 (1×2)	4 (2×2)	4 (4×1)	10
Total Marks	**7**	**8**	**25**	**40**
	7 + 8 = 15			

Internal assessment : 10 Marks

** **L.A.**

Arithmetic

(i) Partnership business —— 1 out of 2 questions : 5×1 marks = 5 marks

Algebra

(i) Variation
(ii) Quadratic equation in one variable } —— 1 out of 2 questions : 3×1 marks = 3 marks

Geometry

(i) Theorems related to tangent to a circle
(ii) Theorems related to similarity } Theorem —— 1 out of 2 questions : 5×1 marks = 5 marks

(iii) Construction of circumcircle and incircle of a triangle } Construction —— 1 out of 2 questions : 5×1 marks = 5 marks

(iv) Application —— 1question : 3×1 marks = 3 marks

Mensuration

(i) Sphere
(ii) Right circular cone } —— 1 out of 2 questions : 4×1 marks = 4 marks

SECOND SUMMATIVE EVALUATION

MODEL QUESTION

TIME: 1 HOUR 30 MINUTES **FULL MARKS: 40**

1. Choose the correct answer : **1×7=7**

(i) If the roots of the equation $9x^2+6x+a=0$ be real and equal than the value of a will be

(a) $-\frac{2}{3}$ (b) $\frac{1}{9}$ (c) 1 (d) 9

(ii) A started a business of fish mongering with Rs.2000. After 2 days, B invested Rs.3000 in that business. He made a contract with A that the two will share the profit on the ratio of their share of capital. If the share of profit is calculated after 10 days of A started the business, then the ratio of profit between A and B will be

(a) 2 : 3 (b) 1 : 5 (c) 5 : 4 (d) 5 : 6

(iii) From the combined gas laws of Boyle and Charles regarding ideal gas we get PV=RT, where R is the universal gas constant. Here, the relation of T and $\frac{1}{P}$ with V is

(a) directly proportional (b) inversely proportional

(c) jointly proportional (d) directly and inversely proportional

(iv) In a circle with radius 5 cm, AB and BC are two chords perpendicular to each other. The length of the chord AC is

(a) 5 cm (b) 2.5 cm (c) 10 cm (d) 5.5 cm

(v) In a circle with 6cm radius P is a point outside the circle. If the length of the tangent from P to a point on the circle Q is 8cm, then the distance from P to the centre of the circle is

(a) 14 cm (b) 36 cm (c) $2\sqrt{7}$ cm (d) 10 cm

(vi) The height of a cone is double the radius. If the radius of the cone is doubled and its height is halved, then the volume of the new cone in contrast to old one will be

(a) equal (b) double (c) 4 times (d) 8 times

(vii) The ratio of the total area of a solid semicircular sphere with the area of its plane surface is

(a) 2:1 (b) 1:2 (c) 1:3 (d) 3:1

2. Answer the following questions: **2×4=8**

(i) If $A \propto B$, then prove $A^3 \propto B^3$

(ii) Fill in the blanks:

(a) If a perpendicular is drawn on the extreme point of the diameter of a circle, then the perpendicular will be a ________ to the circle.

(b) If two circles intersect each other, then the two centres and the point of intersection will be________.

(iii) If the numerical value of the volume of a solid hemisphere and the area of its whole surface are same, what is the numerical value of its radius?

(iv) The length of the radius of the base of a cone is 1.4 cm. and its diagonal height is 2.6 cm. What is the area of its plane?

3. Two friends X and Y started a business with Rs. 1, 10,000 and Rs. 90,000 respectively. They decided to share $\frac{3}{5}$ of the income every month for their work in the ratio of 3:2 and the remaining amount will be shared between them in the ratio of their respective share in the capital. If the shared amount of a month of X in respect of his share in the capital is Rs.6600, what is the total income of that month?

(OR)

Two friends started a business at the beginning of the year with contributions of Rs. 50,000 and Rs. 40,000 respectively. After few months, a third friend invested Rs. 20,000 in that business. At the end of the year the profit was Rs. 31,000. If the total profit of the first two friends was Rs. 27,000, how many months later did the third friend invest in the business? **5**

4. If $a \propto b$ and $b \propto c$, prove that $a^3+b^3+c^3 \propto abc$

(OR)

Solve : $\frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} + \frac{1}{(x-4)(x-5)} = \frac{1}{6}$ **3**

5. Prove that from a point outside a circle if two tangents are drawn to that circle, then the line segments that connect the point outside the circle to the two points where the tangents meet the circle are same forming equal angle at the centre.

(OR)

Prove that the area of a square drawn on the hypotenuse of a right angled triangle is equal to sum of the area of the squares drawn on the other two sides of the triangle. **5**

6. In ΔABC a perpendicular AD is drawn from the vertex A on the side BC. If $\frac{BD}{DA} = \frac{DA}{DC}$ prove that ΔABC is a right angled triangle.

7. Draw a right angled triangle whose length of the hypotenuse is 4.5 cm and length of any other side is 3.5 cm. Draw circum circle of this triangle. (Only traces of construction are required) **5**

(OR)

Draw a circle of radius 3 cm. and draw a tangent on any point on that circle.

8. The lengths of the two adjacent sides of a right angle in a right angled triangle are 8 cm and 6 cm respectively. If the triangle is revolved on the axis of the longer side, then find the area of cuboid that is formed from it.

(OR)

The radius of a solid iron sphere is 12cm. How many conical figures of 2 cm radius and 3 cm height will be formed by melting the iron sphere? **5**

Marks distribution of Third Summative Evaluation/Selection Test

(Summative-III)

Subject	MCQ	V S A		S A	L A **	
	(1×6)	Fill in the blanks 5 out of 6 (1×5)	True or False 5 out of 6 (1×5)	10 out of 12 (2×10)		
Arithmetic	1	1	1	4 (2×2)	5 (5×1)	
Algebra	1	1	1	4 (2×2)	9 (3+3+3)	
Geometry	1	1	1	6 (2×3)	13 (5+3+5)	
Trigonometry	1	1	1	4 (2×2)	11 (3+3+5)	
Mensuration	1	1	1	4 (2×2)	8 (4+4)	
Statistics	1	1	1	2 (2×1)	8 (4+4)	
Total	**6**	**5**	**5**	**20**	**54**	**90**
Marks	**6 + 5 + 5 + 20 = 36**					

**** LA**

Internal Formative Evaluation : 10 Marks

Arithmetic

(i) Simple interest
(ii) Compound interest and uniform rate of increase or decrease
(iii) Partnership business

} 1 out of 2 questions : 5×1 marks = 5 marks

Algebra

(i) Quadratic equation in one variable — 1 out of 2 questions : 3×1 marks = 3 marks
(ii) Variation
(iii) Quadratic Surd
} — 1 out of 2 questions : 3×1 marks = 3 marks
(iv) Ratio and proportion — 1 out of 2 questions : 3×1 marks = 3 marks

Geometry

1 out of 2 theorems : 5×1 marks = 5 marks
Application of theorem for the solution of geometric problems– 1 out of 2 questions : 3×1 marks = 3 marks
Construction : 1 out of 2 questions : 5×1 marks = 5 marks

Trigonometry

(i) Concept of measurement of angle
(ii) Trigonometric Ratio and Trigonometric Identities
(iii) Trigonometric Ratios of complementary angle
} —2 out of 3 questions : 3×2 marks = 6 marks
(iv) Application of Trigonometric Ratios:Heights & Distances– 1out of 2 questions: 5×1marks=5marks

Mensuration

(i) Cuboid
(ii) Right circular cylinder
(iii) Sphere
(iv) Right circular cone
(v) Problems related to different solid objects
} — 2 out of 3 questions : 4×2 marks = 8 marks

Statistics

Mean, Median, Ogive, Mode — 2 out of 3 questions : 4×2 marks = 8 marks

THIRD SUMMATIVE EVALUATION/SELECTION TEST

MODEL QUESTION

TIME: 3 HOURS **FULL MARKS: 90**

1. A. Choose the correct answer (Answer all the questions): **1×6=6**

(i) The total money is kept in three banks in such way that after 1 year the interest in each of the three banks is same. If the annual simple interest of the three banks is 4%, 8% and 12% respectively, the amount kept in the third bank in relation to that of the first bank is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) 3 times

(ii) The roots of the quadratic equation $\frac{1}{a}x^2 - (b + c)\,x + abc = 0\ (a \neq 0)$ are

(a) b and c (b) $\frac{b}{a}$ and $\frac{c}{a}$ (c) ab and ac (d) a and bc

(iii) If one angle of a right angled triangle is α and $\sin 2\alpha = 1$ then which of the following is possible

(a) $\sin \alpha = \cos \alpha$ (b) $\sin \alpha = \tan \alpha$ (c) $\sin \alpha = \sec \alpha$ (d) $\sin \alpha = \operatorname{cosec} \alpha$

(iv) If two circles of radii R and r respectively touch each other externally, then the distance between the two centres is

(a) $R + r$ (b) $R - r$ (c) 0 (d) $2(R+r)$

(v) If the whole surface area of a cube and its volume are equal numerically, then the length of the cube is

(a) 1 unit (b) 2 unit (c) 5 unit (d) 6 unit

(vi) The mode of data 1, 2, 6, 1, 6, 2, 3, 3 is

(a) 1 (b) 2 (c) 3 (d) 1,2,3,6

B. Fill in the blanks (any five): **1×5=5**

(i) If the compound interest at r% per annum is Rs________, then the amount of Rs. 'p' in 'n' years = $p\left(1+\frac{r}{400}\right)^{4n}$

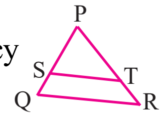
(ii) If a: 4 = b: 10, then 25% of a = ________% of b.

(iii) In a circular system if θ be the value of an angle made by an arc of length 'S' unit subtended at the centre of a circle of radius 'r' unit, then 'S' = ________.

(iv) If two circles touch each other externally then the number of common tangent of the two circles will be___________.

(v) If the height of a cone of radius ‘r’ be ‘r’ unit, then the total volume of 4 cones is equal to the volume of a sphere of _________ radius.

(vi) In the arranged frequency distribution table the class which bears maximum frequency is known as ____________ class.



C. State whether the following statements are True or False: 1×5=5

(i) In a joint partnership if the principal of X is $\frac{1}{3}$ of Y, then the ratio of profit of X and Y are 3 : 1

(ii) If x is inversely proportional to y and y is inversely proportional to z then x is directly proportional to z.

(iii) The value of $(\sin^2 1^\circ + \sin^2 89^\circ)$ is 1.

(iv) In ΔPQR“PQR of the given diagram, $\frac{PS}{SQ} = \frac{PT}{TR}$ If $\angle PST \neq \angle PRQ$ then ΔPQR is an isosceles triangle.

(v) The ratio of the surface area of a sphere and of a hemisphere with radius ‘r’ unit is 4 : 3.

(vi) The value of the median can be derived from the point of intersection of the less than type and greater than type ogives in the graph, but it cannot be derived from one ogive.

2. Answer the following questions: 2×10=20

(i) Every year the number of street accidents decreases by 10% than the previous year. If in 2017 there were 54 street accidents, how many accidents occurred in 2016?

(ii) In how many years Rs. 1000 amounts to Rs.1440 at a compound interest of 20% per annum?

(iii) $x \propto \sqrt{y}$ and $y = a^2$ than $x = a$, y- then express y in terms of x.

(iv) Find the relation of the two roots of the quadratic equation $ax^2 - (a^2+1)x + a = 0 (a \neq 0)$.

(v) The radii of a cone and a cylinder are equal. But the height of the cone is double the cylinder. What will be the ratio of volume of the cone and the cylinder?

(vi) The transversal height of a cone is 4times of the radius of its base. If the area of the curved surface be 16 π sq. m, what is the length of the diameter?

(vii) PQ and RS are two equal and parallel chords of a circle with centre O. The length of each chord is 20 cm. If the radius of the circle is 13 cm, find the distance between the two chords?

(viii) AT is a tangent at a point A on the circle with centre O. The extended portion of the diameter BC intersects the tangent at the point T. If $\angle ABC = 25^\circ$ find the value of $\angle ATB$.

(ix) In the ΔPQR, QR is parallel to ST which intersects PQ at S and PR at T. If PQ = (x+1) unit, PS=3 unit, PR=(x+6) unit and PT =6 unit, then find the value of x.

(x) In the $\tan\theta = \cot\alpha$ $0^\circ < \theta < 90^\circ$, $0^\circ < \alpha < 90^\circ$, then find the value of $\cos(\theta+\alpha)$.

(xi) Find the value of each angle of a right angled isosceles triangle in circular system.

(xii) In some data if the assumed mean a=50, class length h=20, total frequencies $\Sigma f_i = 50$ and $\Sigma f_i u_i = 10$ then find the arithmetic mean where u_i = deviations of the values of the variable with respect to assumed mean for per unit class length.

3. If the difference of 5% per annum simple interest and compound interest for 2 years is Rs 10, calculate the principle.

(OR)

Two friends started a business with Rs. 100,000 and Rs. 150,000 respectively. At the end of the year the total profit is Rs. 50,000. They deposited 20% of their profit to the insurance. Then they equally divided among themselves $\frac{1}{4}$ of the remaining profit. After dividing the profit if they further share the remaining profit in the proportion of their investment, calculate the profit of each friend. **5**

4. Solve : $(\frac{x+5}{x-5})^2 + \frac{x+5}{x-5} - 12 = 0$

(OR)

If multiplying a two digit positive number with its tenth digit number the product is 117 and the unit digit number is three times the tenth digit, then find the number. **3**

5. Simplify : $\frac{3\sqrt{3}}{\sqrt{2}+\sqrt{5}} - \frac{2\sqrt{2}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{2}+\sqrt{3}}$

(OR)

The volume of the sphere is directly proportional to the cube of the length of radius of the sphere. The diameter of a solid lead sphere is 12 cm. Melting this sphere 3 new solid spheres were made. The volume does not change by melting the sphere. If the radii of the two new spheres are 3cm and 4cm respectively, find the length of the third diameter by applying the principle of variations. **3**

6. $a^2 : (yb + zc) = b^2 : (zc + xa) = c^2 : (xa + yb) = 1$

show that $\frac{x}{x+a} + \frac{y}{y+b} + \frac{z}{z+c} = 1$

(OR)

If a, b, c, d are continued proportion, show that $\frac{a^2-b^2}{b^2-c^2} = \frac{b^2-c^2}{c^2-d^2}$ **3**

7. **Prove:** If the area of a square drawn on one side of a triangle is equal to the sum of the area of squares drawn on the other two sides, the angle opposite to the first side is a right angle.

(OR)

Prove that the tangent to a circle at any point on it is perpendicular to the radias passes through the point of contect. **5**

8. From the vertex A of the ΔABC the perpendicular AD on BC intersects BC at D. If $AD^2 = BD.CD$, prove that ΔABC is a right angled triangle and $\angle A = 90^\circ$

(OR)

Draw a circle with centre O and diameter AB. RS is a chord parallel to the tangent PAQ at the point A. Prove that AB is perpendicular bisector of RS. **3**

9. Draw mean proportional of two line segments of 7cm and 3 cm respectively. Find the value of $\sqrt{21}$ from the diagram. (Use construction signs only)

(OR)

Draw an isosceles triangle whose length of base is 5cm and the length of the two equal sides are 6cm each. Draw an in-circle of the triangle. (Use construction signs only) **5**

10. Answer any two questions: 3×2=6

(i) If the difference between the measure of two acute angles of a right angled triangle is 10°, find the value of the two angles in circular system.

(ii) If $\tan\theta = \frac{x}{y}$ show that $\sin\theta - \cos\theta = \frac{x-y}{\sqrt{x^2+y^2}}$

(iii) Prove that, $\cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 78^\circ \cot 60^\circ = \frac{1}{\sqrt{3}}$

11. Answer any two questions: 4×2=8

(i) A right circular cylinder of 28cm diameter contains some water. Three solid iron spheres of equal diameter can be completely immersed into the water. The height of the water increased by 7cm after 3 spheres was immersed into it. Find the diameter of the spheres.

(ii) The diameter of the base of a right circular cone is 21 m and its height is 14 m. Find the cost of painting the outer side of the cone at Rs.1.50 per sq. m.

(iii) 220 kiloliter of water leaked through a hole into the hold of a ship. After sealing the hole, a pump was set into action to pump out the water. The diameter of the pipe of the pump is 20 cm. If the rate of pumping out of water is 350 m per second, calculate how long would the pump be set into action to clear all the water?

12. If one looks from the roof of a house of 200 m height to the opposite house, then the angles of depression of the top and base of the house are 30° and 60° respectively. Calculate the height of the house. 5

(OR)

If the angle of elevation of the sun increases from 30^0 to 45^0, then the length of the shadow of a post decreases by 3m. Find the height of the post.

13. Answer any two questions: **4×2=8**

(i) The ages of 100 persons present in a programme are given in the table below. Calculate the mean age of the 100 persons.

Age	10-20	20-30	30-40	40-50	50-60	60-70
No. of persons	12	08	22	20	18	20

(ii)

Class	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	04	10	15	08	03	05

Develop the cumulative frequency distribution table (greater than type) of the given data and draw an Ogive in a graph paper.

(iii) Find the mode of the data from the frequency distribution given below:

Size no. of the substance	4-8	8-12	12-16	16-20	20-24	24-28	28-32
Frequency	9	10	18	14	10	6	3

Relevant points on quadratic equation

1. If α,β are the roots of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ where a, b, c are real numbers then find the equation whose roots are the receiprocal of α and β.
   Observe the similarities of the coefficients of the previous equation and the new equation.

   e.g., the roots of $x^2 + 2x + 3 = 0$ are α and β

   Here,Coefficient of x^2 is 1,

   Coefficient of x^1 is 2

   and Coefficient of x^0 is 3

   Find the Coefficients of x^2 , x^1 and x^0 of the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

2. If a rational number $\frac{p}{q}$ is a root of a quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ where a, b, c are real numbers then q is a divisor of a and p is a divisor of c .
   Verify the above statement with three examples.

   Find all possible rational roots of $6x^2 + 7x - 3 = 0$ with the help of the above statement

3. • Let α, β be the roots of the quadratic equation $ax^2 + bx + c = 0$ where a, b, c are real numbers
   A polynomial function $f(\alpha, \beta)$ of roots α and β is said to be a symmetric function if $f(\beta, \alpha) = f(\alpha, \beta)$

   e.g., $f(\alpha, \beta) = \alpha^3 + \beta^3 + \alpha^2\beta^2$ is a symmetric funtnon of roots .

   • $\alpha + \beta$ and $\alpha\beta$ are elementary symmetric functions of roots.
   With the help of the values of $\alpha + \beta$ and $\alpha\beta$ we can easily find the values of any symmetric functions of roots. But we can not easily find the values of non symmetric functions of roots

   e.g. Let α, β be the roots of the quadratic equation $6x^2 + 7x + 3 = 0$

   (i) find the value of $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$ and

   (ii) find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha^2}$ with the help of the values of $\alpha + \beta$ and $\alpha\beta$

4. Consider the quadratic polynomial $ax^2 + bx + c$ $(a \neq 0)$ where a, b, c are real numbers. We are not able to factorise the polynomial $ax^2 + bx + c$ into linear factors with real coefficients $ax^2 + bx + c = (dx + e)(fx + g)$, where d, e, f and g are real numbers if there exists a prime p which divides b and c but p does not divide a and p^2 does not divide c

   e.g. $7x^2 + 4x + 2$ is irreducible (factorization is not possible)

Discussion on different areas of life skill development in the curriculum of Mathematics for class X

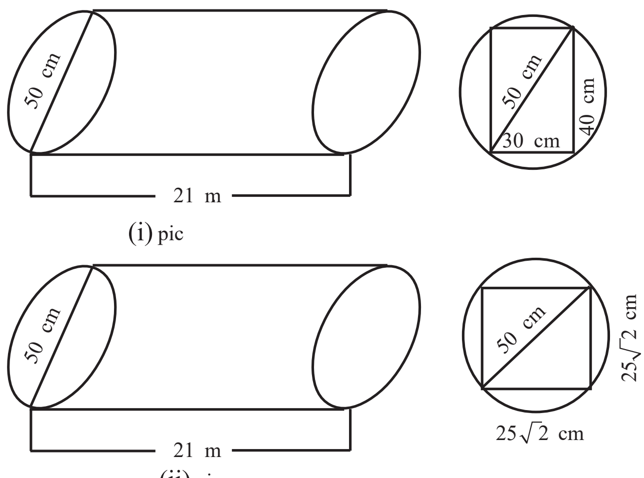
The goal of education is the holistic development of the learner. This is in fact lifelong learning. The experiences gathered from the interactions of daily life integrate with the visions of formal education guide the learner towards the right path of development. But, does the school curriculum provide competence to the learners to face the challenge of adversity even after the completion of school education? Does the curriculum make the learner skillful enough to utilize the minimum opportunities in adverse situations? Needless to say, life skills are required to overcome such challenges of adversities.

Special education is required for the learners to overcome the limitations of school curriculum, face the pressure of within and outside and the several uncertainties of real life, maintain individual identities, develop their innate qualities, make themselves befitting for a fast changing world and to grow up into a sensitive and socially consciousable citizen. Such an education is known as Life skill education which complements school curriculum. Life skill education can be given to the learners for enhancement of their skills. Life skill education helps in cognitive development, enhances effective communicative skills and alleviates the pressure of life.

Various areas of life skill development are explicit in Mathematics curriculum. Some familiar instances of such possible areas are provided here:

Life skill development area

Topic: Real life problem on solid substance



(i) pic

(ii) pic

Two cylindrical wooden logs are shown in the figures (i) and (ii). The length and diameter of each wooden log is 21 m. and 50 cm. respectively. Two cuboids have to be cut out from the logs wasting minimum wood. The unused (or wasted) wood has to be utilized by pasting to make a wooden plank. The volume of the cuboid and that of the wooden plank have to be calculated. The price of the cuboid is high and that of the plank is less.

Now, the question is, what will be the cross-section of the cuboid? Will it be rectangular or square shaped?

Student 1 has cut the log like figure (i), i.e. the cross-section in the shape of a rectangle whose length is 40 cm and breadth 30 cm. (i.e. measurement of length and breadth are different). S/he will calculate the volume of the cuboid and the wooden plank from this data (the volume of glue is not considered).

Student 2 has cut the log like figure (ii), i.e. the cross-section in the shape of a square whose length is $25\sqrt{2}$ cm (35.35 cm approx.). S/he will also calculate the two volumes. Thus, they will realize which of the two methods is profitable.

The life skills that are developed here are—

1. (Wise use of Resources)
2. (Critical Thinking)
3. (Problem Solving)
4. (Decision Making)

Characteristics of Some life skills

Problem Solving

- Ability to identify the problem, i.e. he/she is able to identify those mathematical matters related to the problem
- Ability to write the problem in mathematical language
- Ability to think about the possible ways and also to apply the most appropriate way of solving the roblem
- Ability to analyze logically the way of solving the problem and to practice it

Decision Making

- Ability to know why a decision is being made on a given topic
- Ability to select with utmost confidence the most important and effective method
- Ability to think about the possible advantages to be gained for selecting the method along with the probable disadvantages to be faced
- Ability to verify the decision

Critical Thinking

- Remembering
- Comprehending
- Applying
- Analysing
- Evaluating
- Creating

Wise use of Resources

- Understanding the value of resources and using them appropriately
- After using the resources with proper objective, utilizing the other resources in varied ways appropriately
- Transforming the resources by maintaining their importance
- Adopting appropriate planning for successful implementation of the objectives
- Feeling motivated from the experience gathered in the desired field of activity and achieving success in any other field

Marketable Skill

- Reaction based on diverse circumstances
- Marketing strategies
- Decision making based on circumstances
- Preparation for adopting new approach based on experience

Keeping Records and Planning/Organizing

- Decision can be taken regarding the planning or organizing an activity on the basis of the data recorded and the objectives undertaken
- Apprehending the future perspective of an event based on analysis and comparison
- Knowing about the essence of the matter
- Realizing the errors and mistakes committed and rectifying them
- Knowing about a topic in detail and being aware of its responsibility

Bloom's Action Verbs

Remembering	Understanding	Applying	Analyzing	Evaluating	Creating
Copy	Ask	Act	Advertise	Appraise	Adapt
Define	Associate	Administer	Analyze	Argue	Anticipate
Describing	Cite	Apply	Appraise	Assess	Arrange
Discover	Classify	Articulate	Break Down	Choose	Assemble
Duplicate	Compare	Calculate	Calculate	Compare	Choose
Enumerate	Contrast	Chang	Classify	Conclude	Collaborate
Examine	Convert	Chart	Compare	Consider	Collect
Identify	Demonstrate	Choose	Conclude	Convince	Combine
Label	Describe	Collect	Connect	Criticize	Compile
Listen	Differentiate	Complete	Contrast	Critique	Compose
List	Distinguish	Compute	Correlate	Debate	Construct
Locate	Estimate	Construct	Criticize	Decide	Create
Matching	Explain	Demonstrate	Deduce	Defend	Design
Memorize	Express	Determine	Devise	Discriminate	Develop
Name	Extend	Develop	Diagram	Distinguish	Devise
Observe	Generalize	Discover	Differentiate	Editorialize	Express
Omit	Give Examples	Dramatize	Discriminate	Estimate	Facilitate
Quote	Group	Employ	Dissect	Evaluate	Formulate
Read	Identify	Establish	Distinguish	Find Errors	Generalize
Recall	Illustrate	Experiment	Divide	Grade	Hypothesize
Recite	Indicate	Explain	Estimate	Judge	Imagine
Recognize	Infer	Illustrate	Evaluate	Justify	Infer
Record	Interpret	Interpret	Experiment	Measure	Integrate
Repeat	Judge	Interview	Explain	Order	Intervene
Reproduce	Observe	Judge	Focus	Persuade	Invent
Retell	Order	List	Illustrate	Predict	Justify
Select	Paraphrase	Manipulate	Infer	Rank	Make
State	Predict	Modify	Order	Rate	Manage
Tabulate	Relate	Operate	Organize	Recommend	Modify
Tell	Report	Paint	Outline	Reframe	Negotiate
Visualize	Representing	Practice	Plan	Score	Organize
	Research	Predict	Point-Out	Select	Originate
	Restate	Prepare	Prioritize	Summarize	Plan
	Review	Produce	Question	Support	Prepare
	Rewrite	Record	Select	Test	Produce
	Select	Relate	Separate	Weigh	Propose
	Show	Report	Subdivide		Rearrange
	Summarize	Schedule	Survey		Reorganize
	Trace	Show	Test		Report
	Transform	Simulate			Revise
	Translate	Sketch			Rewrite
		Solve			Role-Play
		Teach			Schematize
		Transfer			Simulate
		Use			Solve
		Write			Speculate
					Structure
					Support
					Validate
					Write

सत्यमेव जयते

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