

**BANGLAR SIKSHA PORTAL**  
**H.S. MATHEMATICS 2022**  
**TOPIC-INDEFINITE INTEGRAL**

**13.02.2022**

মান নির্ণয় কর -

1.  $\int \tan^{-1}\left(\frac{\sin x}{1-\cos x}\right) dx$

Ans.  $I = \int \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx$   
 $= \int \tan^{-1}\left(\cot\frac{x}{2}\right) dx$   
 $= \int \tan^{-1}\tan\left(\frac{\pi}{2}-\frac{x}{2}\right) dx$   
 $= \int\left(\frac{\pi}{2}-\frac{x}{2}\right) dx$   
 $= \frac{\pi x}{2}-\frac{x^2}{4}+c$

2. মান নির্ণয় কর -

$$\int \cos^{-1}\sqrt{\frac{x}{x+a}} dx$$

Ans.  $I = \int \cos^{-1}\sqrt{\frac{a \tan^2 \theta}{a \sec^2 \theta}} \cdot 2a \tan \theta \sec^2 \theta d\theta$

Put  $x = a \tan^2 \theta$

$dx = 2a \tan \theta \sec^2 \theta d\theta$

$$= \int \cos^{-1}(\sin \theta) 2a \tan \theta \sec^2 \theta d\theta$$

$$= 2a \int \left(\frac{\pi}{2} - \theta\right) \tan \theta d(\tan \theta)$$

$$= 2a \cdot \frac{\pi}{2} \int \tan \theta d(\tan \theta) - 2a \int \theta \cdot \tan \theta d(\tan \theta)$$

$$= a\pi \cdot \frac{\tan^2 \theta}{2} - 2a \left[ \theta \cdot \int \tan \theta d(\tan \theta) - \int \left\{ \frac{d}{d\theta}(\theta) \cdot \int \tan \theta d(\tan \theta) \right\} d\theta \right]$$

$$= \frac{a\pi}{2} \tan^2 \theta - 2a \left[ \frac{\theta}{2} \tan^2 \theta - \int \frac{\tan^2 \theta}{2} d\theta \right]$$

$$\begin{aligned}
&= \frac{a\pi}{2} \tan^2 \theta - a\theta \tan^2 \theta + a \int (\sec^2 \theta - 1) d\theta \\
&= \frac{a\pi}{2} \tan^2 \theta - a\theta \tan^2 \theta + a \tan \theta - a\theta + c \\
&= \frac{a\pi}{2} \cdot \frac{x}{a} - a\theta \sec^2 \theta + a \tan \theta + c \\
&= \frac{\pi x}{2} - a \left(1 + \frac{x}{a}\right) \tan^{-1} \sqrt{\frac{x}{a}} + a \sqrt{\frac{x}{a}} + c \\
&= \frac{\pi x}{2} - (a+x) \tan^{-1} \sqrt{\frac{x}{a}} + \sqrt{ax} + c
\end{aligned}$$

3. মান নির্ণয় কর -

$$\int \frac{e^x(1+x)}{\sin^2(xe^x)} dx$$

Ans.

$$\begin{aligned}
\text{Put } xe^x = z &\Rightarrow (e^x + xe^x) dx = dz \\
&\Rightarrow (x+1)e^x dx = dz
\end{aligned}$$

$$\begin{aligned}
I &= \int \frac{dz}{\sin^2 z} = \int \operatorname{cosec}^2 z dz \\
&= -\cot z + c = -\cot(xe^x) + c
\end{aligned}$$

4. মান নির্ণয় কর -

$$\int \frac{x^4 - 1}{x^2 \sqrt{x^4 + x^2 + 1}} dx$$

$$\text{Ans. } I = \int \frac{x - \frac{1}{x^3}}{\sqrt{x^2 + 1 + \frac{1}{x^2}}} dx \quad (\text{dividing numerator and denominator by } x^3)$$

$$I = \int \frac{z dz}{z}$$

$$= \int dz$$

$$= z + c$$

$$= \sqrt{x^2 + 1 + \frac{1}{x^2}} + c$$

$$\text{Put } x^2 + 1 + \frac{1}{x^2} = z^2$$

$$\therefore \left(2x - \frac{2}{x^3}\right) dx = 2z dz$$

$$\therefore \left(x - \frac{1}{x^3}\right) dx = z dz$$

5. মান নির্ণয় কর -

$$\int \frac{dx}{1+e^x}$$

Ans.  $I = \int \frac{e^{-x} dx}{e^{-x} + 1}$  Put  $e^{-x} = z$

$$\Rightarrow -e^x dx = dz$$

$$= -\int \frac{dz}{z+1} = -\ln|z+1| + c$$

$$= -\ln|e^{-x} + 1| + c$$

6. মান নির্ণয় কর -

$$\int \frac{\cos x - \cos 2x}{1 - \cos x} dx$$

Ans.  $I = \int \frac{\cos x - (2 \cos^2 x - 1)}{1 - \cos x} dx$

$$= \int \frac{1 + \cos x - 2 \cos^2 x}{1 - \cos x} dx$$

$$= \int \frac{(1 + 2 \cos x)(1 - \cos x)}{(1 - \cos x)} dx$$

$$= \int (1 + 2 \cos x) dx = x + 2 \sin x + c$$

7. মান নির্ণয় কর -

$$\int \frac{dx}{\sqrt{\frac{2}{3}x^3 - x^2 + \frac{1}{3}}}$$

Ans.  $I = \int \frac{\sqrt{3}}{\sqrt{2x^3 - 3x^2 + 1}} dx$

$$= \sqrt{3} \int \frac{dx}{\sqrt{(x-1)^2(2x+1)}}$$

$$= \sqrt{3} \int \frac{dx}{(x-1)\sqrt{2x+1}}$$

Put  $2x+1 = z^2 \Rightarrow dx = z dz$

$$= \sqrt{3} \int \frac{z dz}{\left(\frac{z^2-1}{2} - 1\right) \cdot z}$$

$$\begin{aligned}
&= 2\sqrt{3} \int \frac{dz}{z^2 - (\sqrt{3})^2} \\
&= 2\sqrt{3} \cdot \frac{1}{2\sqrt{3}} \log_e \left| \frac{z - \sqrt{3}}{z + \sqrt{3}} \right| + c \\
&= \log_e \left| \frac{\sqrt{2x+1} - \sqrt{3}}{\sqrt{2x+1} + \sqrt{3}} \right| + c
\end{aligned}$$

8. মান নির্ণয় কর -

$$\int \sqrt{\tan x} dx$$

Ans.  $I = \int z \frac{2z}{1+z^4} dz$  Put  $\tan x = z^2 \Rightarrow \sec^2 x dx = 2z dz \Rightarrow dx = \frac{2z}{1+z^4} dz$

$$= 2 \int \frac{z^2}{1+z^4} dz$$

$$= 2 \int \frac{dz}{z^2 + \frac{1}{z^2}}$$

$$= \int \frac{\left(1 + \frac{1}{z^2}\right) + \left(1 - \frac{1}{z^2}\right)}{z^2 + \frac{1}{z^2}} dz$$

$$= \int \frac{1 + \frac{1}{z^2}}{\left(z - \frac{1}{z}\right)^2 + 2} dz + \int \frac{1 - \frac{1}{z^2}}{\left(z + \frac{1}{z}\right)^2 - 2} dz$$

$$= \int \frac{d\left(z - \frac{1}{z}\right)}{\left(z - \frac{1}{z}\right)^2 + (\sqrt{2})^2} + \int \frac{d\left(z + \frac{1}{z}\right)}{\left(z + \frac{1}{z}\right)^2 - (\sqrt{2})^2} dz$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{z - \frac{1}{z}}{\sqrt{2}} \right) + \frac{1}{2\sqrt{2}} \log_e \left| \frac{z + \frac{1}{z} - \sqrt{2}}{z + \frac{1}{z} + \sqrt{2}} \right| + c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{z^2 - 1}{z\sqrt{2}} \right) + \frac{1}{2\sqrt{2}} \log_e \left| \frac{z^2 - \sqrt{2}z + 1}{z^2 + \sqrt{2}z + 1} \right| + c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{\tan x - 1}{\sqrt{2} \tan x} \right) + \frac{1}{2\sqrt{2}} \log_e \left| \frac{\tan x - \sqrt{2} \tan x + 1}{\tan x + \sqrt{2} \tan x + 1} \right| + c$$

যেখানে  $c$  হল একটি সমাকলন ধ্রুবক।